Chapter 2

Functions and Their Graphs

Section 2.1

1. $(-1, 3)$

2. $3(-2)^2 - 5(-2) + \frac{1}{(-2)} = 3(4) - 5(-2) - \frac{1}{2}$

$= 12 + 10 - \frac{1}{2}$

$= \frac{43}{2}$ or $21\frac{1}{2}$

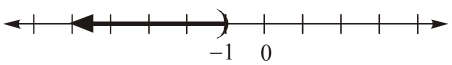
3. We must not allow the denominator to be 0. $x + 4 \neq 0 \Rightarrow x \neq -4$; Domain: $\{x \mid x \neq -4\}$.

4. $3 - 2x > 5$

$-2x > 2$

$x < -1$

Solution set: $\{x \mid x < -1\}$ or $(-\infty, -1)$

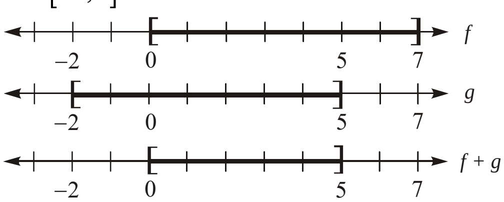


5. independent; dependent

6. range

7. $[0, 5]$

We need the intersection of the intervals $[0, 7]$ and $[-2, 5]$.



8. $\neq$; $f(x)$; $g(x)$

9. $g(x) - f(x)$, or $(g - f)(x)$

10. False; every function is a relation, but not every relation is a function. For example, the relation $x^2 + y^2 = 1$ is not a function.

11. True

12. True

13. False; if the domain is not specified, we assume it is the largest set of real numbers for which the value of *f* is a real number.

14. False; the domain of $f(x) = \frac{x^2 - 4}{x}$ is $\{x \mid x \neq 0\}$.

15. Function
Domain: {Elvis, Colleen, Kaleigh, Marissa}
Range: {Jan. 8, Mar. 15, Sept. 17}

16. Not a function

17. Not a function

18. Function
Domain: {Less than 9^{th} grade, 9^{th}-12^{th} grade, High School Graduate, Some College, College Graduate}
Range: {$18,120, $23,251, $36,055, $45,810, $67,165}

19. Not a function

20. Function
Domain: {–2, –1, 3, 4}
Range: {3, 5, 7, 12}

21. Function
Domain: {1, 2, 3, 4}
Range: {3}

22. Function
Domain: {0, 1, 2, 3}
Range: {–2, 3, 7}

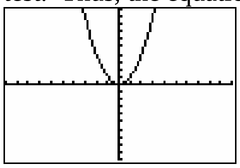
23. Not a function

24. Not a function

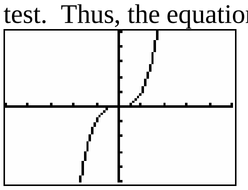
25. Function
Domain: {–2, –1, 0, 1}
Range: {0, 1, 4}

26. Function
Domain: {–2, –1, 0, 1}
Range: {3, 4, 16}

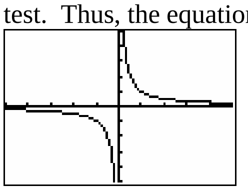
27. Graph $y = x^2$. The graph passes the vertical line test. Thus, the equation represents a function.



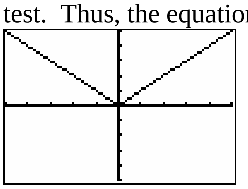
28. Graph $y = x^3$. The graph passes the vertical line test. Thus, the equation represents a function.



29. Graph $y = \frac{1}{x}$. The graph passes the vertical line test. Thus, the equation represents a function.



30. Graph $y = |x|$. The graph passes the vertical line test. Thus, the equation represents a function.



31. $y^2 = 4 - x^2$

Solve for y: $y = \pm\sqrt{4 - x^2}$

For $x = 0, y = \pm 2$. Thus, (0, 2) and (0, –2) are on the graph. This is not a function, since a distinct x corresponds to two different y's.

32. $y = \pm\sqrt{1 - 2x}$

For $x = 0, y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct x corresponds to two different y's.

33. $x = y^2$

Solve for y: $y = \pm\sqrt{x}$

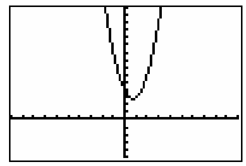
For $x = 1, y = \pm 1$. Thus, (1, 1) and (1, –1) are on the graph. This is not a function, since a distinct x corresponds to two different y's.

34. $x + y^2 = 1$

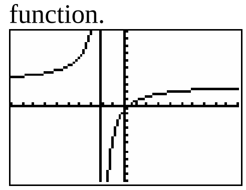
Solve for y: $y = \pm\sqrt{1 - x}$

For $x = 0, y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct x corresponds to two different y's.

35. Graph $y = 2x^2 - 3x + 4$. The graph passes the vertical line test. Thus, the equation represents a function.



36. Graph $y = \frac{3x - 1}{x + 2}$. The graph passes the vertical line test. Thus, the equation represents a function.



37. $2x^2 + 3y^2 = 1$

Solve for y: $2x^2 + 3y^2 = 1$

$$3y^2 = 1 - 2x^2$$

$$y^2 = \frac{1 - 2x^2}{3}$$

$$y = \pm\sqrt{\frac{1 - 2x^2}{3}}$$

For $x = 0, y = \pm\sqrt{\frac{1}{3}}$. Thus, $\left(0, \sqrt{\frac{1}{3}}\right)$ and $\left(0, -\sqrt{\frac{1}{3}}\right)$ are on the graph. This is not a function, since a distinct x corresponds to two different y's.

38. $x^2 - 4y^2 = 1$

Solve for *y*: $x^2 - 4y^2 = 1$

$$4y^2 = x^2 - 1$$
$$y^2 = \frac{x^2-1}{4}$$
$$y = \frac{\pm\sqrt{x^2-1}}{2}$$

For $x = \sqrt{2}, y = \pm\frac{1}{2}$. Thus, $\left(\sqrt{2}, \frac{1}{2}\right)$ and $\left(\sqrt{2}, -\frac{1}{2}\right)$ are on the graph. This is not a function, since a distinct x corresponds to two different y 's.

39. $f(x) = 3x^2 + 2x - 4$

a. $f(0) = 3(0)^2 + 2(0) - 4 = -4$

b. $f(1) = 3(1)^2 + 2(1) - 4 = 3 + 2 - 4 = 1$

c. $f(-1) = 3(-1)^2 + 2(-1) - 4 = 3 - 2 - 4 = -3$

d. $f(-x) = 3(-x)^2 + 2(-x) - 4 = 3x^2 - 2x - 4$

e. $-f(x) = -\left(3x^2 + 2x - 4\right) = -3x^2 - 2x + 4$

f. $f(x+1) = 3(x+1)^2 + 2(x+1) - 4$
$$= 3\left(x^2 + 2x + 1\right) + 2x + 2 - 4$$
$$= 3x^2 + 6x + 3 + 2x + 2 - 4$$
$$= 3x^2 + 8x + 1$$

g. $f(2x) = 3(2x)^2 + 2(2x) - 4 = 12x^2 + 4x - 4$

h. $f(x+h) = 3(x+h)^2 + 2(x+h) - 4$
$$= 3\left(x^2 + 2xh + h^2\right) + 2x + 2h - 4$$
$$= 3x^2 + 6xh + 3h^2 + 2x + 2h - 4$$

40. $f(x) = -2x^2 + x - 1$

a. $f(0) = -2(0)^2 + 0 - 1 = -1$

b. $f(1) = -2(1)^2 + 1 - 1 = -2$

c. $f(-1) = -2(-1)^2 + (-1) - 1 = -4$

d. $f(-x) = -2(-x)^2 + (-x) - 1 = -2x^2 - x - 1$

e. $-f(x) = -\left[-\left(2x^2 + x - 1\right)\right] = 2x^2 - x + 1$

f. $f(x+1) = -2(x+1)^2 + (x+1) - 1$
$$= -2\left(x^2 + 2x + 1\right) + x + 1 - 1$$
$$= -2x^2 - 4x - 2 + x$$
$$= -2x^2 - 3x - 2$$

g. $f(2x) = -2(2x)^2 + (2x) - 1 = -8x^2 + 2x - 1$

h. $f(x+h) = -2(x+h)^2 + (x+h) - 1$
$$= -2\left(x^2 + 2xh + h^2\right) + x + h - 1$$
$$= -2x^2 - 4xh - 2h^2 + x + h - 1$$

41. $f(x) = \frac{x}{x^2+1}$

a. $f(0) = \frac{0}{0^2+1} = \frac{0}{1} = 0$

b. $f(1) = \frac{1}{1^2+1} = \frac{1}{2}$

c. $f(-1) = \frac{-1}{(-1)^2+1} = \frac{-1}{1+1} = -\frac{1}{2}$

d. $f(-x) = \frac{-x}{(-x)^2+1} = \frac{-x}{x^2+1}$

e. $-f(x) = -\left(\frac{x}{x^2+1}\right) = \frac{-x}{x^2+1}$

f. $f(x+1) = \frac{x+1}{(x+1)^2+1}$
$$= \frac{x+1}{x^2+2x+1+1}$$
$$= \frac{x+1}{x^2+2x+2}$$

g. $f(2x) = \frac{2x}{(2x)^2+1} = \frac{2x}{4x^2+1}$

h. $f(x+h) = \frac{x+h}{(x+h)^2+1} = \frac{x+h}{x^2+2xh+h^2+1}$

42. $f(x) = \frac{x^2-1}{x+4}$

a. $f(0) = \frac{0^2-1}{0+4} = \frac{-1}{4} = -\frac{1}{4}$

b. $f(1) = \dfrac{1^2 - 1}{1+4} = \dfrac{0}{5} = 0$

c. $f(-1) = \dfrac{(-1)^2 - 1}{-1+4} = \dfrac{0}{3} = 0$

d. $f(-x) = \dfrac{(-x)^2 - 1}{-x+4} = \dfrac{x^2 - 1}{-x+4}$

e. $-f(x) = -\left(\dfrac{x^2 - 1}{x+4}\right) = \dfrac{1 - x^2}{x+4}$

f. $f(x+1) = \dfrac{(x+1)^2 - 1}{(x+1)+4} = \dfrac{x^2 + 2x + 1 - 1}{x+5}$
$= \dfrac{x^2 + 2x}{x+5}$

g. $f(2x) = \dfrac{(2x)^2 - 1}{2x+4} = \dfrac{4x^2 - 1}{2x+4}$

h. $f(x+h) = \dfrac{(x+h)^2 - 1}{(x+h)+4} = \dfrac{x^2 + 2xh + h^2 - 1}{x+h+4}$

43. $f(x) = |x| + 4$

a. $f(0) = |0| + 4 = 0 + 4 = 4$

b. $f(1) = |1| + 4 = 1 + 4 = 5$

c. $f(-1) = |-1| + 4 = 1 + 4 = 5$

d. $f(-x) = |-x| + 4 = |x| + 4$

e. $-f(x) = -(|x| + 4) = -|x| - 4$

f. $f(x+1) = |x+1| + 4$

g. $f(2x) = |2x| + 4 = 2|x| + 4$

h. $f(x+h) = |x+h| + 4$

44. $f(x) = \sqrt{x^2 + x}$

a. $f(0) = \sqrt{0^2 + 0} = \sqrt{0} = 0$

b. $f(1) = \sqrt{1^2 + 1} = \sqrt{2}$

c. $f(-1) = \sqrt{(-1)^2 + (-1)} = \sqrt{1-1} = \sqrt{0} = 0$

d. $f(-x) = \sqrt{(-x)^2 + (-x)} = \sqrt{x^2 - x}$

e. $-f(x) = -\left(\sqrt{x^2 + x}\right) = -\sqrt{x^2 + x}$

f. $f(x+1) = \sqrt{(x+1)^2 + (x+1)}$
$= \sqrt{x^2 + 2x + 1 + x + 1}$
$= \sqrt{x^2 + 3x + 2}$

g. $f(2x) = \sqrt{(2x)^2 + 2x} = \sqrt{4x^2 + 2x}$

h. $f(x+h) = \sqrt{(x+h)^2 + (x+h)}$
$= \sqrt{x^2 + 2xh + h^2 + x + h}$

45. $f(x) = \dfrac{2x+1}{3x-5}$

a. $f(0) = \dfrac{2(0)+1}{3(0)-5} = \dfrac{0+1}{0-5} = -\dfrac{1}{5}$

b. $f(1) = \dfrac{2(1)+1}{3(1)-5} = \dfrac{2+1}{3-5} = \dfrac{3}{-2} = -\dfrac{3}{2}$

c. $f(-1) = \dfrac{2(-1)+1}{3(-1)-5} = \dfrac{-2+1}{-3-5} = \dfrac{-1}{-8} = \dfrac{1}{8}$

d. $f(-x) = \dfrac{2(-x)+1}{3(-x)-5} = \dfrac{-2x+1}{-3x-5} = \dfrac{2x-1}{3x+5}$

e. $-f(x) = -\left(\dfrac{2x+1}{3x-5}\right) = \dfrac{-2x-1}{3x-5}$

f. $f(x+1) = \dfrac{2(x+1)+1}{3(x+1)-5} = \dfrac{2x+2+1}{3x+3-5} = \dfrac{2x+3}{3x-2}$

g. $f(2x) = \dfrac{2(2x)+1}{3(2x)-5} = \dfrac{4x+1}{6x-5}$

h. $f(x+h) = \dfrac{2(x+h)+1}{3(x+h)-5} = \dfrac{2x+2h+1}{3x+3h-5}$

46. $f(x) = 1 - \dfrac{1}{(x+2)^2}$

a. $f(0) = 1 - \dfrac{1}{(0+2)^2} = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

b. $f(1) = 1 - \dfrac{1}{(1+2)^2} = 1 - \dfrac{1}{9} = \dfrac{8}{9}$

c. $f(-1) = 1 - \frac{1}{(-1+2)^2} = 1 - \frac{1}{1} = 0$

d. $f(-x) = 1 - \frac{1}{(-x+2)^2} = 1 - \frac{1}{(2-x)^2}$

e. $-f(x) = -\left(1 - \frac{1}{(x+2)^2}\right) = \frac{1}{(x+2)^2} - 1$

f. $f(x+1) = 1 - \frac{1}{(x+1+2)^2} = 1 - \frac{1}{(x+3)^2}$

g. $f(2x) = 1 - \frac{1}{(2x+2)^2} = 1 - \frac{1}{4(x+1)^2}$

h. $f(x+h) = 1 - \frac{1}{(x+h+2)^2}$

47. $f(x) = -5x + 4$

Domain: $\{x \mid x \text{ is any real number}\}$

48. $f(x) = x^2 + 2$

Domain: $\{x \mid x \text{ is any real number}\}$

49. $f(x) = \frac{x}{x^2+1}$

Domain: $\{x \mid x \text{ is any real number}\}$

50. $f(x) = \frac{x^2}{x^2+1}$

Domain: $\{x \mid x \text{ is any real number}\}$

51. $g(x) = \frac{x}{x^2-16}$

$x^2 - 16 \neq 0$

$x^2 \neq 16 \Rightarrow x \neq \pm 4$

Domain: $\{x \mid x \neq -4, x \neq 4\}$

52. $h(x) = \frac{2x}{x^2-4}$

$x^2 - 4 \neq 0$

$x^2 \neq 4 \Rightarrow x \neq \pm 2$

Domain: $\{x \mid x \neq -2, x \neq 2\}$

53. $F(x) = \frac{x-2}{x^3+x}$

$x^3 + x \neq 0$

$x(x^2+1) \neq 0$

$x \neq 0, \quad x^2 \neq -1$

Domain: $\{x \mid x \neq 0\}$

54. $G(x) = \frac{x+4}{x^3-4x}$

$x^3 - 4x \neq 0$

$x(x^2-4) \neq 0$

$x \neq 0, \quad x^2 \neq 4$

$x \neq 0, \quad x \neq \pm 2$

Domain: $\{x \mid x \neq 0, x \neq 2, x \neq -2\}$

55. $h(x) = \sqrt{3x-12}$

$3x - 12 \geq 0$

$3x \geq 12$

$x \geq 4$

Domain: $\{x \mid x \geq 4\}$

56. $G(x) = \sqrt{1-x}$

$1 - x \geq 0$

$-x \geq -1$

$x \leq 1$

Domain: $\{x \mid x \leq 1\}$

57. $f(x) = \frac{4}{\sqrt{x-9}}$

$x - 9 > 0$

$x > 9$

Domain: $\{x \mid x > 9\}$

58. $f(x) = \frac{x}{\sqrt{x-4}}$

$x - 4 > 0$

$x > 4$

Domain: $\{x \mid x > 4\}$

59. $p(x)=\sqrt{\dfrac{2}{x-1}}=\dfrac{\sqrt{2}}{\sqrt{x-1}}$

$x-1>0$

$x>1$

Domain: $\{x \mid x>1\}$

60. $q(x)=\sqrt{-x-2}$

$-x-2\ge 0$

$-x\ge 2$

$x\le -2$

Domain: $\{x \mid x\le -2\}$

61. $f(x)=3x+4 \qquad g(x)=2x-3$

a. $(f+g)(x)=3x+4+2x-3=5x+1$

The domain is $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=(3x+4)-(2x-3)$

$=3x+4-2x+3$

$=x+7$

The domain is $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=(3x+4)(2x-3)$

$=6x^2-9x+8x-12$

$=6x^2-x-12$

The domain is $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{3x+4}{2x-3}$

$2x-3\ne 0 \Rightarrow 2x\ne 3 \Rightarrow x\ne \dfrac{3}{2}$

The domain is $\left\{x \,\middle|\, x\ne \dfrac{3}{2}\right\}$.

62. $f(x)=2x+1 \qquad g(x)=3x-2$

a. $(f+g)(x)=2x+1+3x-2=5x-1$

The domain is $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=(2x+1)-(3x-2)$

$=2x+1-3x+2$

$=-x+3$

The domain is $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=(2x+1)(3x-2)$

$=6x^2-4x+3x-2$

$=6x^2-x-2$

The domain is $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{2x+1}{3x-2}$

$3x-2\ne 0$

$3x\ne 2 \Rightarrow x\ne \dfrac{2}{3}$

The domain is $\left\{x \,\middle|\, x\ne \dfrac{2}{3}\right\}$.

63. $f(x)=x-1 \qquad g(x)=2x^2$

a. $(f+g)(x)=x-1+2x^2=2x^2+x-1$

The domain is $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=(x-1)-(2x^2)$

$=x-1-2x^2$

$=-2x^2+x-1$

The domain is $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=(x-1)(2x^2)=2x^3-2x^2$

The domain is $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{x-1}{2x^2}$

The domain is $\{x \mid x\ne 0\}$.

64. $f(x)=2x^2+3 \qquad g(x)=4x^3+1$

a. $(f+g)(x)=2x^2+3+4x^3+1$

$=4x^3+2x^2+4$

The domain is $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=\left(2x^2+3\right)-\left(4x^3+1\right)$

$=2x^2+3-4x^3-1$

$=-4x^3+2x^2+2$

The domain is $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=\left(2x^2+3\right)\left(4x^3+1\right)$

$=8x^5+12x^3+2x^2+3$

The domain is $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{2x^2+3}{4x^3+1}$

$4x^3+1\neq 0$

$4x^3\neq -1$

$x^3\neq -\dfrac{1}{4}\Rightarrow x\neq \sqrt[3]{-\dfrac{1}{4}}=-\dfrac{\sqrt[3]{2}}{2}$

The domain is $\left\{x \,\middle|\, x\neq -\dfrac{\sqrt[3]{2}}{2}\right\}$.

65. $f(x)=\sqrt{x} \qquad g(x)=3x-5$

a. $(f+g)(x)=\sqrt{x}+3x-5$

The domain is $\{x \mid x\geq 0\}$.

b. $(f-g)(x)=\sqrt{x}-(3x-5)=\sqrt{x}-3x+5$

The domain is $\{x \mid x\geq 0\}$.

c. $(f\cdot g)(x)=\sqrt{x}(3x-5)=3x\sqrt{x}-5\sqrt{x}$

The domain is $\{x \mid x\geq 0\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x}}{3x-5}$

$x\geq 0$ and $3x-5\neq 0$

$3x\neq 5\Rightarrow x\neq \dfrac{5}{3}$

The domain is $\left\{x \,\middle|\, x\geq 0 \text{ and } x\neq \dfrac{5}{3}\right\}$.

66. $f(x)=|x| \qquad g(x)=x$

a. $(f+g)(x)=|x|+x$

The domain is $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=|x|-x$

The domain is $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=|x|\cdot x$

The domain is $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{|x|}{x}$

The domain is $\{x \mid x\neq 0\}$.

67. $f(x)=1+\dfrac{1}{x} \qquad g(x)=\dfrac{1}{x}$

a. $(f+g)(x)=1+\dfrac{1}{x}+\dfrac{1}{x}=1+\dfrac{2}{x}$

The domain is $\{x \mid x\neq 0\}$.

b. $(f-g)(x)=1+\dfrac{1}{x}-\dfrac{1}{x}=1$

The domain is $\{x \mid x\neq 0\}$.

c. $(f\cdot g)(x)=\left(1+\dfrac{1}{x}\right)\dfrac{1}{x}=\dfrac{1}{x}+\dfrac{1}{x^2}$

The domain is $\{x \mid x\neq 0\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{1+\frac{1}{x}}{\frac{1}{x}}=\dfrac{\frac{x+1}{x}}{\frac{1}{x}}=\dfrac{x+1}{x}\cdot\dfrac{x}{1}=x+1$

The domain is $\{x \mid x\neq 0\}$.

68. $f(x)=\sqrt{x-2} \qquad g(x)=\sqrt{4-x}$

a. $(f+g)(x)=\sqrt{x-2}+\sqrt{4-x}$

$x-2\geq 0$ and $4-x\geq 0$

$x\geq 2$ and $-x\geq -4$

$x\leq 4$

The domain is $\{x \mid 2\leq x\leq 4\}$.

b. $(f-g)(x)=\sqrt{x-2}-\sqrt{4-x}$

$x-2\geq 0$ and $4-x\geq 0$

$x\geq 2$ and $-x\geq -4$

$x\leq 4$

The domain is $\{x \mid 2\leq x\leq 4\}$.

c. $(f\cdot g)(x)=\left(\sqrt{x-2}\right)\left(\sqrt{4-x}\right)$

$=\sqrt{-x^2+6x-8}$

$x-2\geq 0$ and $4-x\geq 0$

$x\geq 2$ and $-x\geq -4$

$x\leq 4$

The domain is $\{x \mid 2\leq x\leq 4\}$.

d. $\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x-2}}{\sqrt{4-x}}$

$x-2\ge 0$ and $4-x>0$

$x\ge 2$ and $-x>-4$

$x<4$

The domain is $\{x \mid 2\le x<4\}$.

69. $f(x)=\frac{2x+3}{3x-2}$ $g(x)=\frac{4x}{3x-2}$

a. $(f+g)(x)=\frac{2x+3}{3x-2}+\frac{4x}{3x-2}=\frac{2x+3+4x}{3x-2}=\frac{6x+3}{3x-2}$

$3x-2\ne 0$

$3x\ne 2\Rightarrow x\ne\frac{2}{3}$

The domain is $\left\{x \,\middle|\, x\ne\frac{2}{3}\right\}$.

b. $(f-g)(x)=\frac{2x+3}{3x-2}-\frac{4x}{3x-2}=\frac{2x+3-4x}{3x-2}=\frac{-2x+3}{3x-2}$

$3x-2\ne 0$

$3x\ne 2\Rightarrow x\ne\frac{2}{3}$

The domain is $\left\{x \,\middle|\, x\ne\frac{2}{3}\right\}$.

c. $(f\cdot g)(x)=\left(\frac{2x+3}{3x-2}\right)\left(\frac{4x}{3x-2}\right)=\frac{8x^2+12x}{(3x-2)^2}$

$3x-2\ne 0$

$3x\ne 2\Rightarrow x\ne\frac{2}{3}$

The domain is $\left\{x \,\middle|\, x\ne\frac{2}{3}\right\}$.

d. $\left(\frac{f}{g}\right)(x)=\frac{\frac{2x+3}{3x-2}}{\frac{4x}{3x-2}}=\frac{2x+3}{3x-2}\cdot\frac{3x-2}{4x}=\frac{2x+3}{4x}$

$3x-2\ne 0$ and $x\ne 0$

$3x\ne 2$

$x\ne\frac{2}{3}$

The domain is $\left\{x \,\middle|\, x\ne\frac{2}{3} \text{ and } x\ne 0\right\}$.

70. $f(x)=\sqrt{x+1}$ $g(x)=\frac{2}{x}$

a. $(f+g)(x)=\sqrt{x+1}+\frac{2}{x}$

$x+1\ge 0$ and $x\ne 0$

$x\ge -1$

The domain is $\{x \mid x\ge -1, \text{ and } x\ne 0\}$.

b. $(f-g)(x)=\sqrt{x+1}-\frac{2}{x}$

$x+1\ge 0$ and $x\ne 0$

$x\ge -1$

The domain is $\{x \mid x\ge -1, \text{ and } x\ne 0\}$.

c. $(f\cdot g)(x)=\sqrt{x+1}\cdot\frac{2}{x}=\frac{2\sqrt{x+1}}{x}$

$x+1\ge 0$ and $x\ne 0$

$x\ge -1$

The domain is $\{x \mid x\ge -1, \text{ and } x\ne 0\}$.

d. $\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x+1}}{\frac{2}{x}}=\frac{x\sqrt{x+1}}{2}$

$x+1\ge 0$ and $x\ne 0$

$x\ge -1$

The domain is $\{x \mid x\ge -1, \text{ and } x\ne 0\}$.

71. $f(x)=3x+1$ $(f+g)(x)=6-\frac{1}{2}x$

$6-\frac{1}{2}x=3x+1+g(x)$

$5-\frac{7}{2}x=g(x)$

$g(x)=5-\frac{7}{2}x$

72. $f(x)=\dfrac{1}{x}$ $\quad \left(\dfrac{f}{g}\right)(x)=\dfrac{x+1}{x^2-x}$

$$\frac{x+1}{x^2-x}=\frac{\frac{1}{x}}{g(x)}$$

$$g(x)=\frac{\frac{1}{x}}{\frac{x+1}{x^2-x}}=\frac{1}{x}\cdot\frac{x^2-x}{x+1}$$

$$=\frac{1}{x}\cdot\frac{x(x-1)}{x+1}=\frac{x-1}{x+1}$$

73. $f(x)=4x+3$

$$\frac{f(x+h)-f(x)}{h}=\frac{4(x+h)+3-4x-3}{h}$$

$$=\frac{4x+4h+3-4x-3}{h}$$

$$=\frac{4h}{h}=4$$

74. $f(x)=-3x+1$

$$\frac{f(x+h)-f(x)}{h}=\frac{-3(x+h)+1-(-3x+1)}{h}$$

$$=\frac{-3x-3h+1+3x-1}{h}$$

$$=\frac{-3h}{h}=-3$$

75. $f(x)=x^2-x+4$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{(x+h)^2-(x+h)+4-(x^2-x+4)}{h}$$

$$=\frac{x^2+2xh+h^2-x-h+4-x^2+x-4}{h}$$

$$=\frac{2xh+h^2-h}{h}$$

$$=2x+h-1$$

76. $f(x)=x^2+5x-1$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{(x+h)^2+5(x+h)-1-(x^2+5x-1)}{h}$$

$$=\frac{x^2+2xh+h^2+5x+5h-1-x^2-5x+1}{h}$$

$$=\frac{2xh+h^2+5h}{h}$$

$$=2x+h+5$$

77. $f(x)=x^3-2$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{(x+h)^3-2-(x^3-2)}{h}$$

$$=\frac{x^3+3x^2h+3xh^2+h^3-2-x^3+2}{h}$$

$$=\frac{3x^2h+3xh^2+h^3}{h}$$

$$=3x^2+3xh+h^2$$

78. $f(x)=\dfrac{1}{x+3}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{x+h+3}-\frac{1}{x+3}}{h}$$

$$=\frac{\frac{x+3-(x+3+h)}{(x+h+3)(x+3)}}{h}$$

$$=\left(\frac{x+3-x-3-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$

$$=\left(\frac{-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$

$$=-\frac{1}{(x+h+3)(x+3)}$$

79. $f(x) = 2x^3 + Ax^2 + 4x - 5$ and $f(2) = 5$

$$f(2) = 2(2)^3 + A(2)^2 + 4(2) - 5$$
$$5 = 16 + 4A + 8 - 5$$
$$5 = 4A + 19$$
$$-14 = 4A$$
$$A = -\frac{7}{2}$$

80. $f(x) = 3x^2 - Bx + 4$ and $f(-1) = 12$:

$$f(-1) = 3(-1)^2 - B(-1) + 4$$
$$12 = 3 + B + 4$$
$$B = 5$$

81. $f(x) = \dfrac{3x+8}{2x-A}$ and $f(0) = 2$

$$f(0) = \frac{3(0)+8}{2(0)-A}$$
$$2 = \frac{8}{-A}$$
$$-2A = 8$$
$$A = -4$$

82. $f(x) = \dfrac{2x-B}{3x+4}$ and $f(2) = \dfrac{1}{2}$

$$f(2) = \frac{2(2)-B}{3(2)+4}$$
$$\frac{1}{2} = \frac{4-B}{10}$$
$$5 = 4 - B$$
$$B = -1$$

83. $f(x) = \dfrac{2x-A}{x-3}$ and $f(4) = 0$

$$f(4) = \frac{2(4)-A}{4-3}$$
$$0 = \frac{8-A}{1}$$
$$0 = 8 - A$$
$$A = 8$$

f is undefined when $x = 3$.

84. $f(x) = \dfrac{x-B}{x-A}$, $f(2) = 0$ and $f(1)$ is undefined

$$1 - A = 0 \quad \Rightarrow \quad A = 1$$
$$f(2) = \frac{2-B}{2-1}$$
$$0 = \frac{2-B}{1}$$
$$0 = 2 - B$$
$$B = 2$$

85. Let x represent the length of the rectangle.

Then, $\dfrac{x}{2}$ represents the width of the rectangle since the length is twice the width.
The function for the area is:

$$A(x) = x \cdot \frac{x}{2} = \frac{x^2}{2} = \frac{1}{2}x^2$$

86. Let x represent the length of one of the two equal sides.
The function for the area is:

$$A(x) = \frac{1}{2} \cdot x \cdot x = \frac{1}{2}x^2$$

87. Let x represent the number of hours worked.
The function for the gross salary is: $G(x) = 10x$

88. Let x represent the number of items sold.
The function for the gross salary is:
$G(x) = 10x + 100$

89. **a.**
$$H(1) = 20 - 4.9(1)^2$$
$$= 20 - 4.9$$
$$= 15.1 \text{ meters}$$
$$H(1.1) = 20 - 4.9(1.1)^2 = 20 - 4.9(1.21)$$
$$= 20 - 5.929$$
$$= 14.071 \text{ meters}$$
$$H(1.2) = 20 - 4.9(1.2)^2$$
$$= 20 - 4.9(1.44)$$
$$= 20 - 7.056$$
$$= 12.944 \text{ meters}$$
$$H(1.3) = 20 - 4.9(1.3)^2$$
$$= 20 - 4.9(1.69)$$
$$= 20 - 8.281$$
$$= 11.719 \text{ meters}$$

b. $H(x) = 15$:

$15 = 20 - 4.9x^2$

$-5 = -4.9x^2$

$x^2 \approx 1.0204$

$x \approx 1.01$ seconds

$H(x) = 10$:

$10 = 20 - 4.9x^2$

$-10 = -4.9x^2$

$x^2 \approx 2.0408$

$x \approx 1.43$ seconds

$H(x) = 5$:

$5 = 20 - 4.9x^2$

$-15 = -4.9x^2$

$x^2 \approx 3.0612$

$x \approx 1.75$ seconds

c. $H(x) = 0$

$0 = 20 - 4.9x^2$

$-20 = -4.9x^2$

$x^2 \approx 4.0816$

$x \approx 2.02$ seconds

90. a. $H(1) = 20 - 13(1)^2 = 20 - 13 = 7$ meters

$H(1.1) = 20 - 13(1.1)^2 = 20 - 13(1.21)$
$= 20 - 15.73 = 4.27$ meters

$H(1.2) = 20 - 13(1.2)^2 = 20 - 13(1.44)$
$= 20 - 18.72 = 1.28$ meters

b. $H(x) = 15$

$15 = 20 - 13x^2$

$-5 = -13x^2$

$x^2 \approx 0.3846$

$x \approx 0.62$ seconds

$H(x) = 10$

$10 = 20 - 13x^2$

$-10 = -13x^2$

$x^2 \approx 0.7692$

$x \approx 0.88$ seconds

$H(x) = 5$

$5 = 20 - 13x^2$

$-15 = -13x^2$

$x^2 \approx 1.1538$

$x \approx 1.07$ seconds

c. $H(x) = 0$

$0 = 20 - 13x^2$

$-20 = -13x^2$

$x^2 \approx 1.5385$

$x \approx 1.24$ seconds

91. $C(x) = 100 + \frac{x}{10} + \frac{36,000}{x}$

a. $C(500) = 100 + \frac{500}{10} + \frac{36,000}{500}$
$= 100 + 50 + 72$
$= \$222$

b. $C(450) = 100 + \frac{450}{10} + \frac{36,000}{450}$
$= 100 + 45 + 80$
$= \$225$

c. $C(600) = 100 + \frac{600}{10} + \frac{36,000}{600}$
$= 100 + 60 + 60$
$= \$220$

d. $C(400) = 100 + \frac{400}{10} + \frac{36,000}{400}$
$= 100 + 40 + 90$
$= \$230$

92. $A(x) = 4x\sqrt{1 - x^2}$

a. $A\left(\frac{1}{3}\right) = 4 \cdot \frac{1}{3}\sqrt{1 - \left(\frac{1}{3}\right)^2} = \frac{4}{3}\sqrt{\frac{8}{9}} = \frac{4}{3} \cdot \frac{2\sqrt{2}}{3}$
$= \frac{8\sqrt{2}}{9} \approx 1.26 \text{ ft}^2$

b. $A\left(\frac{1}{2}\right) = 4 \cdot \frac{1}{2}\sqrt{1 - \left(\frac{1}{2}\right)^2} = 2\sqrt{\frac{3}{4}} = 2 \cdot \frac{\sqrt{3}}{2}$
$= \sqrt{3} \approx 1.73 \text{ ft}^2$

c. $A\left(\frac{2}{3}\right)=4\cdot\frac{2}{3}\sqrt{1-\left(\frac{2}{3}\right)^2}=\frac{8}{3}\sqrt{\frac{5}{9}}=\frac{8}{3}\cdot\frac{\sqrt{5}}{3}$

$=\frac{8\sqrt{5}}{9}\approx 1.99 \text{ ft}^2$

93. $R(x)=\left(\frac{L}{P}\right)(x)=\frac{L(x)}{P(x)}$

94. $T(x)=(V+P)(x)=V(x)+P(x)$

95. $H(x)=(P\cdot I)(x)=P(x)\cdot I(x)$

96. $N(x)=(I-T)(x)=I(x)-T(x)$

97. a. $h(x)=2x$

$h(a+b)=2(a+b)=2a+2b$

$=h(a)+h(b)$

$h(x)=2x$ has the property.

b. $g(x)=x^2$

$g(a+b)=(a+b)^2=a^2+2ab+b^2$

Since

$a^2+2ab+b^2\neq a^2+b^2=g(a)+g(b)$,

$g(x)=x^2$ does not have the property.

c. $F(x)=5x-2$

$F(a+b)=5(a+b)-2=5a+5b-2$

Since

$5a+5b-2\neq 5a-2+5b-2=F(a)+F(b)$,

$F(x)=5x-2$ does not have the property.

d. $G(x)=\frac{1}{x}$

$G(a+b)=\frac{1}{a+b}\neq\frac{1}{a}+\frac{1}{b}=G(a)+G(b)$

$G(x)=\frac{1}{x}$ does not have the property.

98. No, $x=-1$ is not in the domain of g, but it is in the domain of f.

99. Answers will vary.

Section 2.2

1. $x^2+4y^2=16$

x-intercepts:

$x^2+4(0)^2=16$

$x^2=16$

$x=\pm4\Rightarrow(-4,0),(4,0)$

y-intercepts:

$(0)^2+4y^2=16$

$4y^2=16$

$y^2=4$

$y=\pm2\Rightarrow(0,-2),(0,2)$

2. False; $x=2y-2$

$-2=2y-2$

$0=2y$

$0=y$

The point $(-2,0)$ is on the graph.

3. vertical

4. $f(5)=-3$

5. $f(x)=ax^2+4$

$a(-1)^2+4=2\Rightarrow a=-2$

6. False; it would fail the vertical line test.

7. False; e.g. $y=\frac{1}{x}$.

8. True

9. a. $f(0)=3$ since $(0,3)$ is on the graph.
$f(-6)=-3$ since $(-6,-3)$ is on the graph.

b. $f(6)=0$ since $(6,0)$ is on the graph.
$f(11)=1$ since $(11,1)$ is on the graph.

c. $f(3)$ is positive since $f(3)\approx 3.7$.

d. $f(-4)$ is negative since $f(-4)\approx -1$.

e. $f(x)=0$ when $x=-3$, $x=6$, and $x=10$.

f. $f(x)>0$ when $-3<x<6$, and $10<x\leq 11$.

g. The domain of f is $\{x \mid -6 \le x \le 11\}$ or $[-6, 11]$.

h. The range of f is $\{y \mid -3 \le y \le 4\}$ or $[-3, 4]$.

i. The x-intercepts are (–3, 0), (6, 0), and (10, 0).

j. The y-intercept is (0, 3).

k. The line $y = \frac{1}{2}$ intersects the graph 3 times.

l. The line $x = 5$ intersects the graph 1 time.

m. $f(x) = 3$ when $x = 0$ and $x = 4$.

n. $f(x) = -2$ when $x = -5$ and $x = 8$.

10. a. $f(0) = 0$ since $(0, 0)$ is on the graph.
$f(6) = 0$ since $(6, 0)$ is on the graph.

b. $f(2) = -2$ since $(2, -2)$ is on the graph.
$f(-2) = 1$ since $(-2, 1)$ is on the graph.

c. $f(3)$ is negative since $f(3) \approx -1$.

d. $f(-1)$ is positive since $f(-1) \approx 1.0$.

e. $f(x) = 0$ when $x = 0$, $x = 4$, and $x = 6$.

f. $f(x) < 0$ when $0 < x < 4$.

g. The domain of f is $\{x \mid -4 \le x \le 6\}$ or $[-4, 6]$.

h. The range of f is $\{y \mid -2 \le y \le 3\}$ or $[-2, 3]$.

i. The x-intercepts are (0, 0), (4, 0), and (6, 0).

j. The y-intercept is (0, 0).

k. The line $y = -1$ intersects the graph 2 times.

l. The line $x = 1$ intersects the graph 1 time.

m. $f(x) = 3$ when $x = 5$.

n. $f(x) = -2$ when $x = 2$.

11. Not a function since vertical lines will intersect the graph in more than one point.

12. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y > 0\}$

b. Intercepts: (0,1)

c. None

13. Function

a. Domain: $\{x \mid -\pi \le x \le \pi\}$;
Range: $\{y \mid -1 \le y \le 1\}$

b. Intercepts: $\left(-\frac{\pi}{2}, 0\right)$, $\left(\frac{\pi}{2}, 0\right)$, (0,1)

c. Symmetry about y-axis.

14. Function

a. Domain: $\{x \mid -\pi \le x \le \pi\}$;
Range: $\{y \mid -1 \le y \le 1\}$

b. Intercepts: $(-\pi, 0)$, $(\pi, 0)$, $(0, 0)$

c. Symmetry about the origin.

15. Not a function since vertical lines will intersect the graph in more than one point.

16. Not a function since vertical lines will intersect the graph in more than one point.

17. Function

a. Domain: $\{x \mid x > 0\}$;
Range: $\{y \mid y \text{ is any real number}\}$

b. Intercepts: (1, 0)

c. None

18. Function

a. Domain: $\{x \mid 0 \le x \le 4\}$;
Range: $\{y \mid 0 \le y \le 3\}$

b. Intercepts: (0, 0)

c. None

19. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y \le 2\}$

b. Intercepts: (–3, 0), (3, 0), (0,2)

c. Symmetry about y-axis.

20. Function

a. Domain: $\{x \mid x \geq -3\}$;

Range: $\{y \mid y \geq 0\}$

b. Intercepts: (–3, 0), (2,0), (0,2)

c. None

21. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \geq -3\}$

b. Intercepts: (1, 0), (3,0), (0,9)

c. None

22. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \leq 5\}$

b. Intercepts: (–1, 0), (2,0), (0,4)

c. None

23. $f(x) = 2x^2 - x - 1$

a. $f(-1) = 2(-1)^2 - (-1) - 1 = 2$

The point $(-1, 2)$ is on the graph of *f*.

b. $f(-2) = 2(-2)^2 - (-2) - 1 = 9$

The point $(-2, 9)$ is on the graph of *f*.

c. Solve for x:

$-1 = 2x^2 - x - 1$

$0 = 2x^2 - x$

$0 = x(2x - 1) \Rightarrow x = 0, x = \frac{1}{2}$

(0, –1) and $\left(\frac{1}{2}, -1\right)$ are on the graph of f.

d. The domain of f is: $\{x \mid x \text{ is any real number}\}$.

e. *x*-intercepts:

$f(x) = 0 \Rightarrow 2x^2 - x - 1 = 0$

$(2x + 1)(x - 1) = 0 \Rightarrow x = -\frac{1}{2}, x = 1$

$\left(-\frac{1}{2}, 0\right)$ and $(1, 0)$

f. *y*-intercept:

$f(0) = 2(0)^2 - 0 - 1 = -1 \Rightarrow (0, -1)$

24. $f(x) = -3x^2 + 5x$

a. $f(-1) = -3(-1)^2 + 5(-1) \neq 2$

The point $(-1, 2)$ is not on the graph of *f*.

b. $f(-2) = -3(-2)^2 + 5(-2) = -22$

The point $(-2, -22)$ is on the graph of *f*.

c. Solve for x:

$-2 = -3x^2 + 5x \Rightarrow 3x^2 - 5x - 2 = 0$

$(3x + 1)(x - 2) = 0 \Rightarrow x = -\frac{1}{3}, x = 2$

(2, –2) and $\left(-\frac{1}{3}, -2\right)$ on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. *x*-intercepts:

$f(x) = 0 \Rightarrow -3x^2 + 5x = 0$

$x(-3x + 5) = 0 \Rightarrow x = 0, x = \frac{5}{3}$

$(0, 0)$ and $\left(\frac{5}{3}, 0\right)$

f. *y*-intercept:

$f(0) = -3(0)^2 + 5(0) = 0 \Rightarrow (0, 0)$

25. $f(x) = \dfrac{x + 2}{x - 6}$

a. $f(3) = \dfrac{3 + 2}{3 - 6} = -\dfrac{5}{3} \neq 14$

The point $(3, 14)$ is not on the graph of *f*.

b. $f(4) = \dfrac{4 + 2}{4 - 6} = \dfrac{6}{-2} = -3$

The point $(4, -3)$ is on the graph of *f*.

c. Solve for x:

$2 = \dfrac{x + 2}{x - 6}$

$2x - 12 = x + 2$

$x = 14$

(14, 2) is a point on the graph of f.

d. The domain of *f* is $\{x \mid x \neq 6\}$.

e. *x*-intercepts:

$f(x)=0 \Rightarrow \dfrac{x+2}{x-6}=0$

$x+2=0 \Rightarrow x=-2 \Rightarrow (-2,0)$

f. *y*-intercept: $f(0)=\dfrac{0+2}{0-6}=-\dfrac{1}{3} \Rightarrow \left(0,-\dfrac{1}{3}\right)$

26. $f(x)=\dfrac{x^2+2}{x+4}$

a. $f(1)=\dfrac{1^2+2}{1+4}=\dfrac{3}{5}$

The point $\left(1,\dfrac{3}{5}\right)$ is on the graph of *f*.

b. $f(0)=\dfrac{0^2+2}{0+4}=\dfrac{2}{4}=\dfrac{1}{2}$

The point $\left(0,\dfrac{1}{2}\right)$ is on the graph of *f*.

c. Solve for x:

$\dfrac{1}{2}=\dfrac{x^2+2}{x+4} \Rightarrow x+4=2x^2+4$

$0=2x^2-x$

$x(2x-1)=0 \Rightarrow x=0 \text{ or } x=\dfrac{1}{2}$

$\left(0,\dfrac{1}{2}\right)$ and $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ are on the graph of f.

d. The domain of *f* is $\{x \mid x \neq -4\}$.

e. *x*-intercepts:

$f(x)=0 \Rightarrow \dfrac{x^2+2}{x+4}=0 \Rightarrow x^2+2=0$

This is impossible, so there are no x-intercepts.

f. *y*-intercept:

$f(0)=\dfrac{0^2+2}{0+4}=\dfrac{2}{4}=\dfrac{1}{2} \Rightarrow \left(0,\dfrac{1}{2}\right)$

27. $f(x)=\dfrac{2x^2}{x^4+1}$

a. $f(-1)=\dfrac{2(-1)^2}{(-1)^4+1}=\dfrac{2}{2}=1$

The point $(-1,1)$ is on the graph of *f*.

b. $f(2)=\dfrac{2(2)^2}{(2)^4+1}=\dfrac{8}{17}$

The point $\left(2,\dfrac{8}{17}\right)$ is on the graph of *f*.

c. Solve for x:

$1=\dfrac{2x^2}{x^4+1}$

$x^4+1=2x^2$

$x^4-2x^2+1=0$

$(x^2-1)^2=0$

$x^2-1=0 \Rightarrow x=\pm 1$

(1,1) and (–1,1) are on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. *x*-intercept:

$f(x)=0 \Rightarrow \dfrac{2x^2}{x^4+1}=0$

$2x^2=0 \Rightarrow x=0 \Rightarrow (0,0)$

f. *y*-intercept:

$f(0)=\dfrac{2(0)^2}{0^4+1}=\dfrac{0}{0+1}=0 \Rightarrow (0,0)$

28. $f(x)=\dfrac{2x}{x-2}$

a. $f\left(\dfrac{1}{2}\right)=\dfrac{2\left(\dfrac{1}{2}\right)}{\dfrac{1}{2}-2}=\dfrac{1}{-\dfrac{3}{2}}=-\dfrac{2}{3}$

The point $\left(\dfrac{1}{2},-\dfrac{2}{3}\right)$ is on the graph of *f*.

b. $f(4)=\dfrac{2(4)}{4-2}=\dfrac{8}{2}=4$

The point $(4,4)$ is on the graph of *f*.

c. Solve for x:

$1=\dfrac{2x}{x-2} \Rightarrow x-2=2x \Rightarrow -2=x$

(–2,1) is a point on the graph of f.

d. The domain of *f* is $\{x \mid x \neq 2\}$.

e. x-intercept:

$$f(x)=0 \Rightarrow \frac{2x}{x-2}=0 \Rightarrow 2x=0$$
$$\Rightarrow x=0 \Rightarrow (0,0)$$

f. y-intercept: $f(0)=\frac{0}{0-2}=0 \Rightarrow (0,0)$

29. $h(x)=\frac{-32x^2}{130^2}+x$

a. $h(100)=\frac{-32(100)^2}{130^2}+100$
$$=\frac{-320,000}{16,900}+100 \approx 81.07 \text{ feet}$$

b. $h(300)=\frac{-32(300)^2}{130^2}+300$
$$=\frac{-2,880,000}{16,900}+300 \approx 129.59 \text{ feet}$$

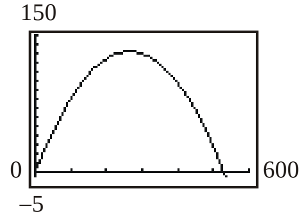
c. $h(500)=\frac{-32(500)^2}{130^2}+500$
$$=\frac{-8,000,000}{16,900}+500 \approx 26.63 \text{ feet}$$

d. Solving $h(x)=\frac{-32x^2}{130^2}+x=0$

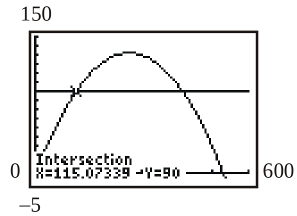
$$\frac{-32x^2}{130^2}+x=0$$
$$x\left(\frac{-32x}{130^2}+1\right)=0$$
$$x=0 \quad \text{or} \quad \frac{-32x}{130^2}+1=0$$
$$1=\frac{32x}{130^2}$$
$$130^2=32x$$
$$x=\frac{130^2}{32}=528.125 \text{ feet}$$

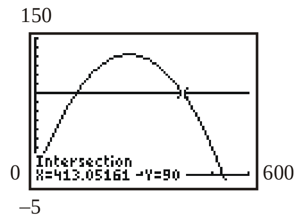
Therefore, the golf ball travels 528.125 feet.

e. $y_1=\frac{-32x^2}{130^2}+x$



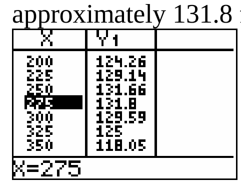
f. Use INTERSECT on the graphs of $y_1=\frac{-32x^2}{130^2}+x$ and $y_2=90$.



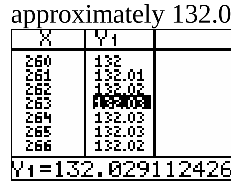


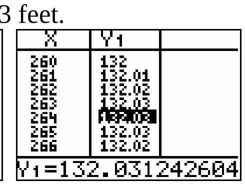
The ball reaches a height of 90 feet twice. The first time is when the ball has traveled approximately 115 feet, and the second time is when the ball has traveled about 413 feet.

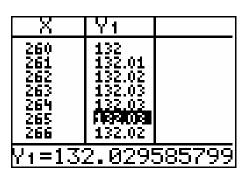
g. The ball travels approximately 275 feet before it reaches its maximum height of approximately 131.8 feet.



h. The ball travels approximately 264 feet before it reaches its maximum height of approximately 132.03 feet.







30. $A(x) = 4x\sqrt{1-x^2}$

a. Domain of $A(x) = 4x\sqrt{1-x^2}$; we know that x must be greater than or equal to zero, since x represents a length. We also need $1-x^2 \ge 0$, since this expression occurs under a square root. In fact, to avoid Area = 0, we require

$x > 0$ and $1-x^2 > 0$.

Solve: $1-x^2 > 0$

$(1+x)(1-x) > 0$

Case1: $1+x > 0$ and $1-x > 0$

$x > -1$ and $x < 1$

(i.e. $-1 < x < 1$)

Case2: $1+x < 0$ and $1-x < 0$

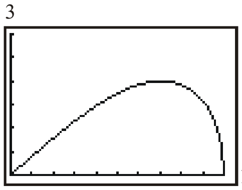
$x < -1$ and $x > 1$

(which is impossible)

Therefore the domain of A is $\{x \mid 0 < x < 1\}$.

b. Graphing $A(x) = 4x\sqrt{1-x^2}$

3



0 1

0

c. When x = 0.7 feet, the cross-sectional area is maximized at approximately 1.9996 square feet. Therefore, the length of the base of the beam should be 1.4 feet in order to maximize the cross-sectional area.

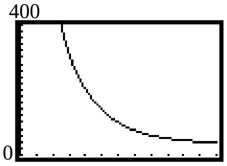
X	Y1	
.3	1.1447	
.4	1.4664	
.5	1.7321	
.6	1.92	
.7	1.9996	
.8	1.92	
.9	1.5692	

X=.7

31. $C(x) = 100 + \dfrac{x}{10} + \dfrac{36000}{x}$

a. Graphing:

400



0 600

200

b. TblStart = 0; ΔTbl = 50

X	Y1	
0	ERROR	
50	825	
100	470	
150	355	
200	300	
250	269	
300	250	

Y1=100+X/10+360…

c. The cost per passenger is minimized to about $220 when the ground speed is roughly 600 miles per hour.

X	Y1	
450	225	
500	222	
550	220.45	
600	220	
650	220.38	
700	221.43	
750	223	

X=600

32. $W(h) = m\left(\dfrac{4000}{4000+h}\right)^2$

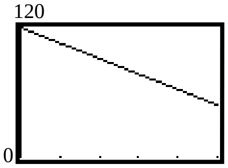
a. $h = 14110$ feet ≈ 2.67 miles;

$W(2.67) = 120\left(\dfrac{4000}{4000+2.67}\right)^2 \approx 119.84$

On Pike's Peak, Amy will weigh about 119.84 pounds.

b. Graphing:

120



0 5

119.5

c. Create a TABLE:

X	Y1	
0	120	
.5	119.97	
1	119.94	
1.5	119.91	
2	119.88	
2.5	119.85	
3	119.82	

X=0

X	Y1	
2	119.88	
2.5	119.85	
3	119.82	
3.5	119.79	
4	119.76	
4.5	119.73	
5	119.7	

X=5

The weight W will vary from 120 pounds to about 119.7 pounds.

d. By refining the table, Amy will weigh 119.95 lbs at a height of about 0.8 miles (4224 feet).

X	Y_1
.5	119.97
.6	119.96
.7	119.96
.8	119.95
.9	119.95
1	119.94
1.1	119.93

X=.8

e. Yes, 4224 feet is reasonable.

33. Answers will vary. From a graph, the domain can be found by visually locating the x-values for which the graph is defined. The range can be found in a similar fashion by visually locating the y-values for which the function is defined.

If an equation is given, the domain can be found by locating any restricted values and removing them from the set of real numbers. The range can be found by using known properties of the graph of the equation, or estimated by means of a table of values.

34. The graph of a function can have any number of x-intercepts.

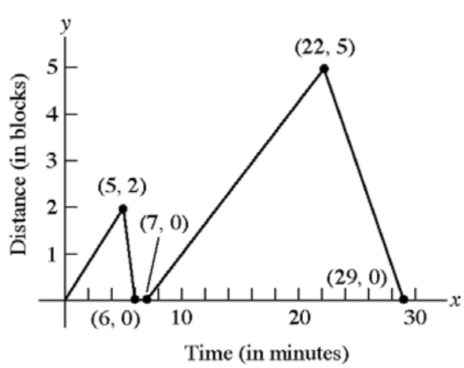
35. The graph of a function can have at most one y-intercept.

36. Yes, the graph of a single point is the graph of a function since it would pass the vertical line test. The equation of such a function would be something like the following:
$f(x) = 2$, where $x = 7$.

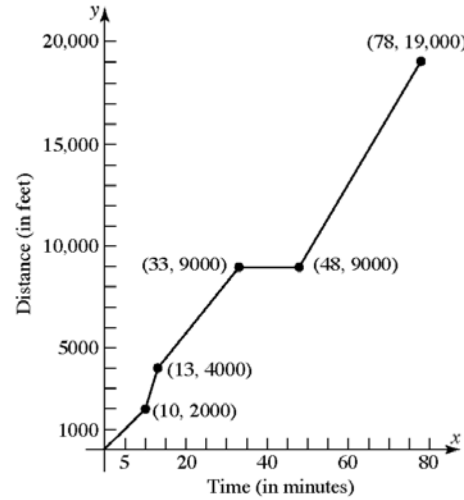
37. (a) III; (b) IV; (c) I; (d) V; (e) II

38. (a) II; (b) V; (c) IV; (d) III; (e) I

39.



40.



41. a. 2 hours elapsed; Kevin was between 0 and 3 miles from home.

b. 0.5 hours elapsed; Kevin was 3 miles from home.

c. 0.3 hours elapsed; Kevin was between 0 and 3 miles from home.

d. 0.2 hours elapsed; Kevin was at home.

e. 0.9 hours elapsed; Kevin was between 0 and 2.8 miles from home.

f. 0.3 hours elapsed; Kevin was 2.8 miles from home.

g. 1.1 hours elapsed; Kevin was between 0 and 2.8 miles from home.

h. The farthest distance Kevin is from home is 3 miles.

i. Kevin returned home 2 times.

42. a. Michael travels fastest between 7 and 7.4 minutes. That is, $(7, 7.4)$.

b. Michael's speed is zero between 4.2 and 6 minutes. That is, $(4.2, 6)$.

c. Between 0 and 2 minutes, Michael's speed increased from 0 to 30 miles/hour.

d. Between 4.2 and 6 minutes, Michael was stopped.

e. Between 7 and 7.4 minutes, Michael was traveling at a steady rate of 50 miles/hour.

f. Michael's speed is constant between 2 and 4 minutes, between 4.2 and 6 minutes, between 7 and 7.4 minutes, and between 7.6 and 8 minutes. That is, on the intervals $(2,4)$, $(4.2,6)$, $(7,7.4)$, and $(7.6,8)$.

43. Answers (graphs) will vary. Points of the form (5, y) and of the form (x, 0) cannot be on the graph of the function.

44. The only such function is $f(x)=0$ because it is the only function for which $f(x)=-f(x)$. Any other such graph would fail the vertical line test.

Section 2.3

1. $2 < x < 5$

2. $\text{slope} = \dfrac{\Delta y}{\Delta x} = \dfrac{8-3}{3-(-2)} = \dfrac{5}{5} = 1$

3. x-axis: $y \to -y$

$(-y) = 5x^2 - 1$

$-y = 5x^2 - 1$

$y = -5x^2 + 1$ different

y-axis: $x \to -x$

$y = 5(-x)^2 - 1$

$y = 5x^2 - 1$ same

origin: $x \to -x$ and $y \to -y$

$(-y) = 5(-x)^2 - 1$

$-y = 5x^2 - 1$

$y = -5x^2 + 1$ different

The equation has symmetry with respect to the y-axis only.

4. $y - y_1 = m(x - x_1)$

$y - (-2) = 5(x-3)$

$y + 2 = 5(x-3)$

5. $y = x^2 - 9$

x-intercepts:

$0 = x^2 - 9$

$x^2 = 9 \to x = \pm 3$

y-intercept:

$y = (0)^2 - 9 = -9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.

6. increasing

7. even; odd

8. True

9. True

10. False; odd functions are symmetric with respect to the origin. Even functions are symmetric with respect to the y-axis.

11. Yes

12. No, it is increasing.

13. No, it only increases on (5, 10).

14. Yes

15. f is increasing on the intervals $(-8,-2)$, $(0,2)$, $(5,\infty)$.

16. f is decreasing on the intervals: $(-\infty,-8)$, $(-2,0)$, $(2,5)$.

17. Yes. The local maximum at $x = 2$ is 10.

18. No. There is a local minimum at $x = 5$; the local minimum is 0.

19. f has local maxima at $x = -2$ and $x = 2$. The local maxima are 6 and 10, respectively.

20. f has local minima at $x = -8$, $x = 0$ and $x = 5$. The local minima are –4, 0, and 0, respectively.

21. a. Intercepts: (–2, 0), (2, 0), and (0, 3).

b. Domain: $\{x \mid -4 \le x \le 4\}$;

Range: $\{y \mid 0 \le y \le 3\}$.

c. Increasing: (–2, 0) and (2, 4);
Decreasing: (–4, –2) and (0, 2).

d. Since the graph is symmetric with respect to the *y*-axis, the function is even.

22. a. Intercepts: (–1, 0), (1, 0), and (0, 2).

b. Domain: $\{x \mid -3 \le x \le 3\}$;

Range: $\{y \mid 0 \le y \le 3\}$.

c. Increasing: (–1, 0) and (1, 3);
Decreasing: (–3, –1) and (0, 1).

d. Since the graph is symmetric with respect to the *y*-axis, the function is even.

23. a. Intercepts: (0, 1).

b. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y > 0\}$.

c. Increasing: $(-\infty, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the *y*-axis or the origin, the function is neither even nor odd.

24. a. Intercepts: (1, 0).

b. Domain: $\{x \mid x > 0\}$;

Range: $\{y \mid y \text{ is any real number}\}$.

c. Increasing: $(0, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the *y*-axis or the origin, the function is neither even nor odd.

25. a. Intercepts: $(-\pi, 0)$, $(\pi, 0)$, and $(0, 0)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$;

Range: $\{y \mid -1 \le y \le 1\}$.

c. Increasing: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$;

Decreasing: $\left(-\pi, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \pi\right)$.

d. Since the graph is symmetric with respect to the origin, the function is odd.

26. a. Intercepts: $\left(-\frac{\pi}{2}, 0\right)$, $\left(\frac{\pi}{2}, 0\right)$, and $(0, 1)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$;

Range: $\{y \mid -1 \le y \le 1\}$.

c. Increasing: $(-\pi, 0)$; Decreasing: $(0, \pi)$.

d. Since the graph is symmetric with respect to the *y*-axis, the function is even.

27. a. Intercepts: $\left(\frac{1}{2}, 0\right)$, $\left(\frac{5}{2}, 0\right)$, and $\left(0, \frac{1}{2}\right)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$;

Range: $\{y \mid -1 \le y \le 2\}$.

c. Increasing: $(2, 3)$; Decreasing: $(-1, 1)$;
Constant: $(-3, -1)$ and $(1, 2)$

d. Since the graph is not symmetric with respect to the *y*-axis or the origin, the function is neither even nor odd.

28. a. Intercepts: $(-2.3, 0)$, $(3, 0)$, and $(0, 1)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$;

Range: $\{y \mid -2 \le y \le 2\}$.

c. Increasing: $(-3, -2)$ and $(0, 2)$;
Decreasing: $(2, 3)$; Constant: $(-2, 0)$.

d. Since the graph is not symmetric with respect to the *y*-axis or the origin, the function is neither even nor odd.

29. a. f has a local maximum of 3 at $x = 0$.

b. f has a local minimum of 0 at both $x = -2$ and $x = 2$.

30. a. f has a local maximum of 2 at $x = 0$.

b. f has a local minimum of 0 at both $x = -1$ and $x = 1$.

31. a. f has a local maximum of 1 at $x = \frac{\pi}{2}$.

b. f has a local minimum of –1 at $x = -\frac{\pi}{2}$.

32. **a.** f has a local maximum of 1 at $x = 0$.

b. f has a local minimum of –1 at $x = -\pi$ and at $x = \pi$.

33. $f(x) = 4x^3$

$f(-x) = 4(-x)^3 = -4x^3 = -f(x)$

Therefore, f is odd.

34. $f(x) = 2x^4 - x^2$

$f(-x) = 2(-x)^4 - (-x)^2 = 2x^4 - x^2 = f(x)$

Therefore, f is even.

35. $g(x) = -3x^2 - 5$

$g(-x) = -3(-x)^2 - 5 = -3x^2 - 5 = g(x)$

Therefore, g is even.

36. $h(x) = 3x^3 + 5$

$h(-x) = 3(-x)^3 + 5 = -3x^3 + 5$

h is neither even nor odd.

37. $F(x) = \sqrt[3]{x}$

$F(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} = -F(x)$

Therefore, F is odd.

38. $G(x) = \sqrt{x}$

$G(-x) = \sqrt{-x}$

G is neither even nor odd.

39. $f(x) = x + |x|$

$f(-x) = -x + |-x| = -x + |x|$

f is neither even nor odd.

40. $f(x) = \sqrt[3]{2x^2 + 1}$

$f(-x) = \sqrt[3]{2(-x)^2 + 1} = \sqrt[3]{2x^2 + 1} = f(x)$

Therefore, f is even.

41. $g(x) = \dfrac{1}{x^2}$

$g(-x) = \dfrac{1}{(-x)^2} = \dfrac{1}{x^2} = g(x)$

Therefore, g is even.

42. $h(x) = \dfrac{x}{x^2 - 1}$

$h(-x) = \dfrac{-x}{(-x)^2 - 1} = \dfrac{-x}{x^2 - 1} = -h(x)$

Therefore, h is odd.

43. $h(x) = \dfrac{-x^3}{3x^2 - 9}$

$h(-x) = \dfrac{-(-x)^3}{3(-x)^2 - 9} = \dfrac{x^3}{3x^2 - 9} = -h(x)$

Therefore, h is odd.

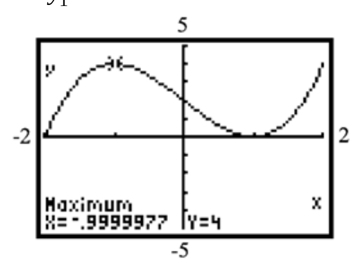
44. $F(x) = \dfrac{2x}{|x|}$

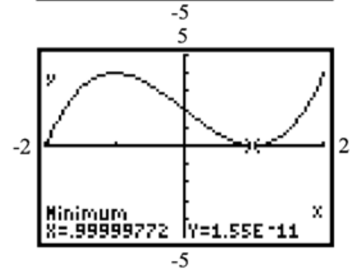
$F(-x) = \dfrac{2(-x)}{|-x|} = \dfrac{-2x}{|x|} = -F(x)$

Therefore, F is odd.

45. $f(x) = x^3 - 3x + 2$ on the interval $(-2, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = x^3 - 3x + 2$.





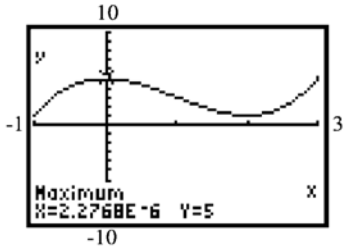
local maximum at: $(-1, 4)$;

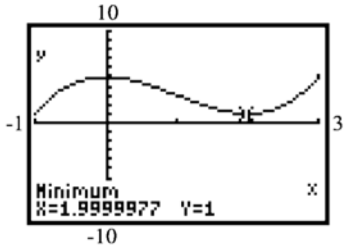
local minimum at: $(1, 0)$

f is increasing on: $(-2, -1)$ and $(1, 2)$;

f is decreasing on: $(-1, 1)$

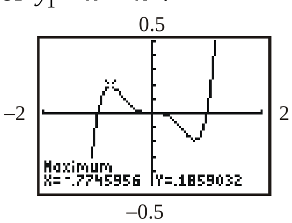
46. $f(x) = x^3 - 3x^2 + 5$ on the interval $(-1, 3)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = x^3 - 3x^2 + 5$.

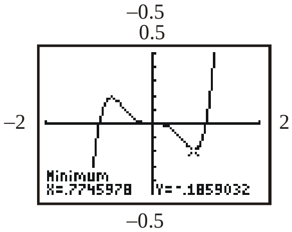




local maximum at: $(0, 5)$;
local minimum at: $(2, 1)$
f is increasing on: $(-1, 0)$ and $(2, 3)$;
f is decreasing on: $(0, 2)$

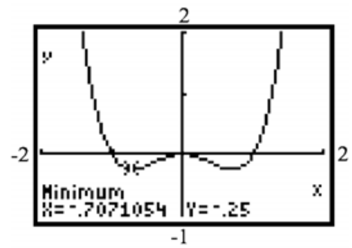
47. $f(x) = x^5 - x^3$ on the interval $(-2, 2)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = x^5 - x^3$.

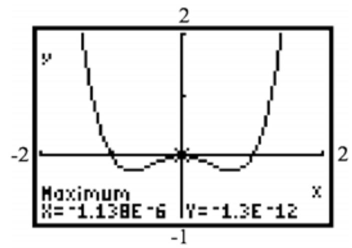


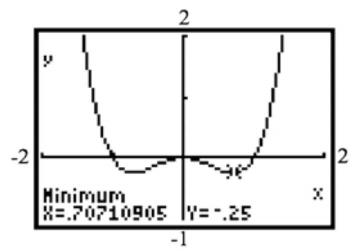


local maximum at: $(-0.77, 0.19)$;
local minimum at: $(0.77, -0.19)$;
f is increasing on: $(-2, -0.77)$ and $(0.77, 2)$;
f is decreasing on: $(-0.77, 0.77)$

48. $f(x) = x^4 - x^2$ on the interval $(-2, 2)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = x^4 - x^2$.

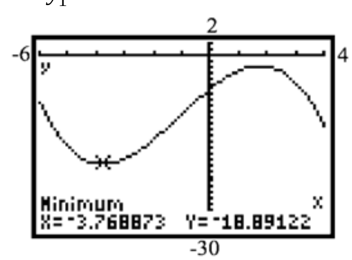


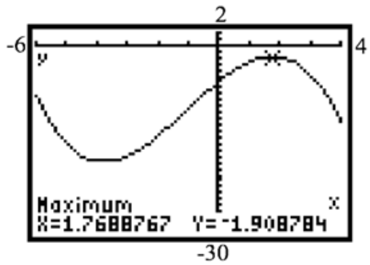




local maximum at: $(0, 0)$;
local minimum at: $(-0.71, -0.25)$, $(0.71, -0.25)$
f is increasing on: $(-0.71, 0)$ and $(0.71, 2)$;
f is decreasing on: $(-2, -0.71)$ and $(0, 0.71)$

49. $f(x) = -0.2x^3 - 0.6x^2 + 4x - 6$ on the interval $(-6, 4)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.2x^3 - 0.6x^2 + 4x - 6$.





local maximum at: $(1.77, -1.91)$;

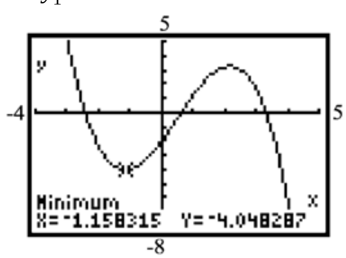
local minimum at: $(-3.77, -18.89)$

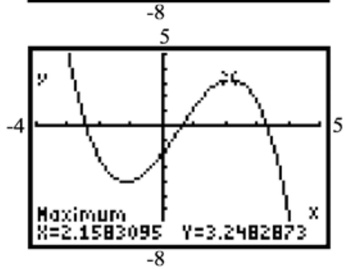
f is increasing on: $(-3.77, 1.77)$;

f is decreasing on: $(-6, -3.77)$ and $(1.77, 4)$

50. $f(x) = -0.4x^3 + 0.6x^2 + 3x - 2$ on the interval $(-4, 5)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^3 + 0.6x^2 + 3x - 2$.





local maximum at: $(2.16, 3.25)$;

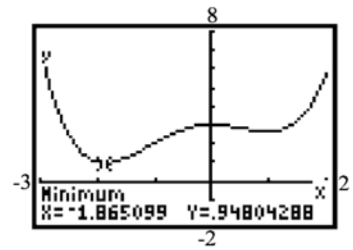
local minimum at: $(-1.16, -4.05)$

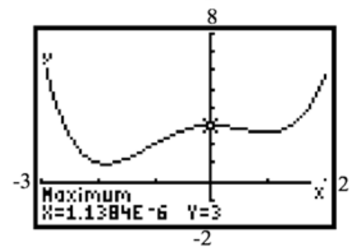
f is increasing on: $(-1.16, 2.16)$;

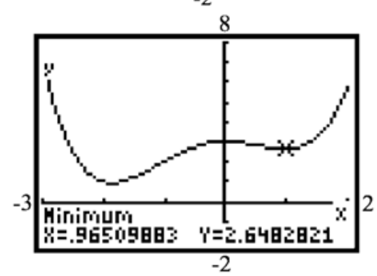
f is decreasing on: $(-4, -1.16)$ and $(2.16, 5)$

51. $f(x) = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$ on the interval $(-3, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$.







local maximum at: $(0, 3)$;

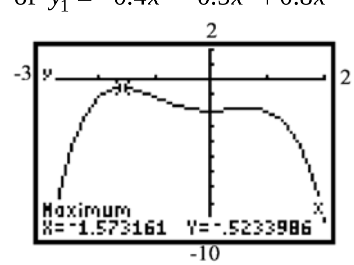
local minimum at: $(-1.87, 0.95)$, $(0.97, 2.65)$

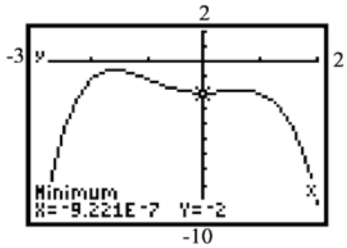
f is increasing on: $(-1.87, 0)$ and $(0.97, 2)$;

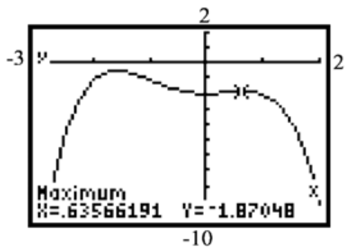
f is decreasing on: $(-3, -1.87)$ and $(0, 0.97)$

52. $f(x) = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$ on the interval $(-3, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$.







local maximum at: $(-1.57,-0.52)$, $(0.64,-1.87)$; local minimum at: $(0,-2)$

f is increasing on: $(-3,-1.57)$ and $(0,0.64)$;

f is decreasing on: $(-1.57,0)$ and $(0.64,2)$

53. $f(x)=-2x^2+4$

a. Average rate of change of f from $x=0$ to $x=2$

$$\frac{f(2)-f(0)}{2-0}=\frac{\left(-2(2)^2+4\right)-\left(-2(0)^2+4\right)}{2}=\frac{(-4)-(4)}{2}=\frac{-8}{2}=-4$$

b. Average rate of change of f from $x = 1$ to $x = 3$:

$$\frac{f(3)-f(1)}{3-1}=\frac{\left(-2(3)^2+4\right)-\left(-2(1)^2+4\right)}{2}=\frac{(-14)-(2)}{2}=\frac{-16}{2}=-8$$

c. Average rate of change of f from $x = 1$ to $x = 4$:

$$\frac{f(4)-f(1)}{4-1}=\frac{\left(-2(4)^2+4\right)-\left(-2(1)^2+4\right)}{3}=\frac{(-28)-(2)}{3}=\frac{-30}{3}=-10$$

54. $f(x)=-x^3+1$

a. Average rate of change of f from $x = 0$ to $x = 2$:

$$\frac{f(2)-f(0)}{2-0}=\frac{\left(-(2)^3+1\right)-\left(-(0)^3+1\right)}{2}=\frac{-7-1}{2}=\frac{-8}{2}=-4$$

b. Average rate of change of f from $x = 1$ to $x = 3$:

$$\frac{f(3)-f(1)}{3-1}=\frac{\left(-(3)^3+1\right)-\left(-(1)^3+1\right)}{2}=\frac{-26-(0)}{2}=\frac{-26}{2}=-13$$

c. Average rate of change of f from $x = -1$ to $x = 1$:

$$\frac{f(1)-f(-1)}{1-(-1)}=\frac{\left(-(1)^3+1\right)-\left(-(-1)^3+1\right)}{2}=\frac{0-2}{2}=\frac{-2}{2}=-1$$

55. $g(x)=x^3-2x+1$

a. Average rate of change of g from $x=-3$ to $x=-2$:

$$\frac{g(-2)-g(-3)}{-2-(-3)}$$
$$=\frac{\left[(-2)^3-2(-2)+1\right]-\left[(-3)^3-2(-3)+1\right]}{1}$$
$$=\frac{(-3)-(-20)}{1}=\frac{17}{1}$$
$$=17$$

b. Average rate of change of g from $x=-1$ to $x=1$:

$$\frac{g(1)-g(-1)}{1-(-1)}$$
$$=\frac{\left[(1)^3-2(1)+1\right]-\left[(-1)^3-2(-1)+1\right]}{2}$$
$$=\frac{(0)-(2)}{2}=\frac{-2}{2}$$
$$=-1$$

c. Average rate of change of g from $x=1$ to $x=3$:

$$\frac{g(3)-g(1)}{3-1}$$
$$=\frac{\left[(3)^3-2(3)+1\right]-\left[(1)^3-2(1)+1\right]}{2}$$
$$=\frac{(22)-(0)}{2}=\frac{22}{2}$$
$$=11$$

56. $h(x)=x^2-2x+3$

a. Average rate of change of h from $x=-1$ to $x=1$:

$$\frac{h(1)-h(-1)}{1-(-1)}$$
$$=\frac{\left[(1)^2-2(1)+3\right]-\left[(-1)^2-2(-1)+3\right]}{2}$$
$$=\frac{(2)-(6)}{2}=\frac{-4}{2}$$
$$=-2$$

b. Average rate of change of h from $x=0$ to $x=2$:

$$\frac{h(2)-h(0)}{2-0}$$
$$=\frac{\left[(2)^2-2(2)+3\right]-\left[(0)^2-2(0)+3\right]}{2}$$
$$=\frac{(3)-(3)}{2}=\frac{0}{2}$$
$$=0$$

c. Average rate of change of h from $x=2$ to $x=5$:

$$\frac{h(5)-h(2)}{5-2}$$
$$=\frac{\left[(5)^2-2(5)+3\right]-\left[(2)^2-2(2)+3\right]}{3}$$
$$=\frac{(18)-(3)}{3}=\frac{15}{3}$$
$$=5$$

57. $f(x)=5x-2$

a. Average rate of change of f from 1 to x:

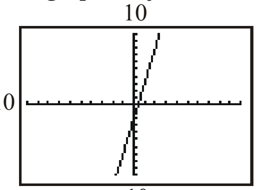
$$\frac{f(x)-f(1)}{x-1}=\frac{(5x-2)-(5(1)-2)}{x-1}$$
$$=\frac{5x-2-3}{x-1}=\frac{5x-5}{x-1}$$
$$=\frac{5(x-1)}{x-1}$$
$$=5$$

b. The average rate of change of f from 1 to x is a constant 5. Therefore, the average rate of change of f from 1 to 3 is 5. The slope of the secant line joining $(1, f(1))$ and $(3, f(3))$ is 5.

c. We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-3=5(x-1)$$
$$y-3=5x-5$$
$$y=5x-2$$

d. The secant line coincides with the function so the graph only shows one line.



58. $f(x)=-4x+1$

a. Average rate of change of f from 2 to x:

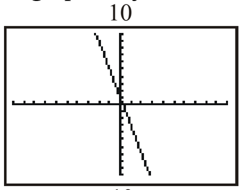
$$\frac{f(x)-f(2)}{x-2}=\frac{(-4x+1)-(-4(2)+1)}{x-2}$$
$$=\frac{-4x+1-(-7)}{x-2}=\frac{-4x+8}{x-2}$$
$$=\frac{-4(x-2)}{x-2}$$
$$=-4$$

b. The average rate of change of f from 2 to x is given by -4. Therefore, the average rate of change of f from 2 to 5 is -4. The slope of the secant line joining $(2, f(2))$ and $(5, f(5))$ is -4.

c. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - (-7) = -4(x - 2)$$
$$y + 7 = -4x + 8$$
$$y = -4x + 1$$

d. The secant line coincides with the function so the graph only shows one line.



59. $g(x) = x^2 - 2$

a. Average rate of change of g from -2 to x:

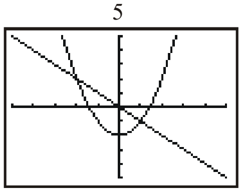
$$\frac{g(x) - g(-2)}{x - (-2)} = \frac{\left[x^2 - 2\right] - \left[(-2)^2 - 2\right]}{x + 2}$$
$$= \frac{(x^2 - 2) - (2)}{x + 2} = \frac{x^2 - 4}{x + 2}$$
$$= \frac{(x + 2)(x - 2)}{x + 2} = x - 2$$

b. The average rate of change of g from -2 to x is given by $x - 2$. Therefore, the average rate of change of g from -2 to 1 is $1 - 2 = -1$. The slope of the secant line joining $(-2, g(-2))$ and $(1, g(1))$ is -1.

c. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 2 = -1(x - (-2))$$
$$y - 2 = -x - 2$$
$$y = -x$$

d. The graph below shows the graph of g along with the secant line $y = -x$.



60. $g(x) = x^2 + 1$

a. Average rate of change of g from -1 to x:

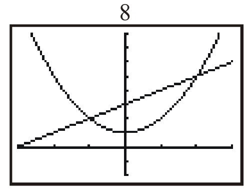
$$\frac{g(x) - g(-1)}{x - (-1)} = \frac{\left[x^2 + 1\right] - \left[(-1)^2 + 1\right]}{x + 1}$$
$$= \frac{(x^2 + 1) - (2)}{x + 1} = \frac{x^2 - 1}{x + 1}$$
$$= \frac{(x - 1)(x + 1)}{x + 1} = x - 1$$

b. The average rate of change of g from -1 to x is given by $x - 1$. Therefore, the average rate of change of g from -1 to 2 is $2 - 1 = 1$. The slope of the secant line joining $(-1, g(-1))$ and $(2, g(2))$ is 1.

c. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 2 = 1(x - (-1))$$
$$y - 2 = x + 1$$
$$y = x + 3$$

d. The graph below shows the graph of g along with the secant line $y = x + 3$.



61. $h(x) = x^2 - 2x$

a. Average rate of change of h from 2 to x:

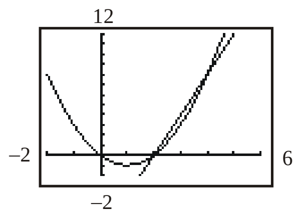
$$\frac{h(x) - h(2)}{x - 2} = \frac{\left[x^2 - 2x\right] - \left[(2)^2 - 2(2)\right]}{x - 2}$$
$$= \frac{(x^2 - 2x) - (0)}{x - 2} = \frac{x^2 - 2x}{x - 2}$$
$$= \frac{x(x - 2)}{x - 2} = x$$

b. The average rate of change of h from 2 to x is given by x. Therefore, the average rate of change of h from 2 to 4 is 4. The slope of the secant line joining $(2, h(2))$ and $(4, h(4))$ is 4.

c. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 0 = 4(x - 2)$$
$$y = 4x - 8$$

d. The graph below shows the graph of h along with the secant line $y = 4x - 8$.



62. $h(x) = -2x^2 + x$

a. Average rate of change from 0 to x:

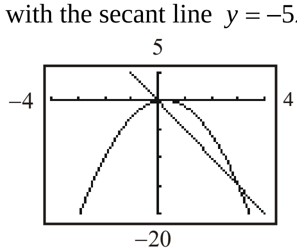
$$\frac{h(x) - h(0)}{x - 0} = \frac{\left[-2x^2 + x\right] - \left[-2(0)^2 + 0\right]}{x}$$
$$= \frac{\left(-2x^2 + x\right) - (0)}{x} = \frac{-2x^2 + x}{x}$$
$$= \frac{x(-2x + 1)}{x} = -2x + 1$$

b. The average rate of change of h from 0 to x is given by $-2x + 1$. Therefore, the average rate of change of h from 0 to 3 is $-2(3) + 1 = -5$. The slope of the secant line joining $(0, h(0))$ and $(3, h(3))$ is -5.

c. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 0 = -5(x - 0)$$
$$y = -5x$$

d. The graph below shows the graph of h along with the secant line $y = -5x$.



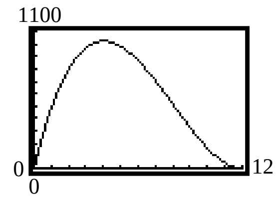
63. a. length = $24 - 2x$; width = $24 - 2x$; height = x

$$V(x) = x(24 - 2x)(24 - 2x) = x(24 - 2x)^2$$

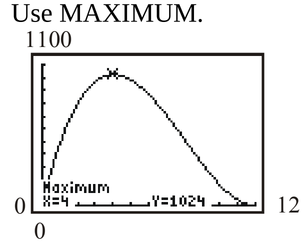
b. $V(3) = 3(24 - 2(3))^2 = 3(18)^2$
$= 3(324) = 972$ cu.in.

c. $V(10) = 10(24 - 2(10))^2 = 10(4)^2$
$= 10(16) = 160$ cu.in.

d. $y_1 = x(24 - 2x)^2$



Use MAXIMUM.



The volume is largest when $x = 4$ inches.

64. a. Let A = amount of material, x = length of the base, h = height, and V = volume.

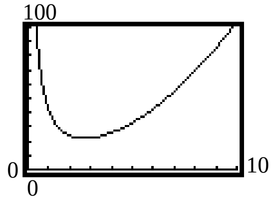
$$V = x^2 h = 10 \Rightarrow h = \frac{10}{x^2}$$

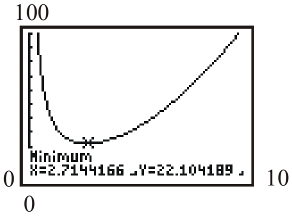
$$\text{Total Area } A = \left(\text{Area}_{\text{base}}\right) + (4)\left(\text{Area}_{\text{side}}\right)$$
$$= x^2 + 4xh$$
$$= x^2 + 4x\left(\frac{10}{x^2}\right)$$
$$= x^2 + \frac{40}{x}$$
$$A(x) = x^2 + \frac{40}{x}$$

b. $A(1) = 1^2 + \frac{40}{1} = 1 + 40 = 41 \text{ ft}^2$

c. $A(2) = 2^2 + \frac{40}{2} = 4 + 20 = 24 \text{ ft}^2$

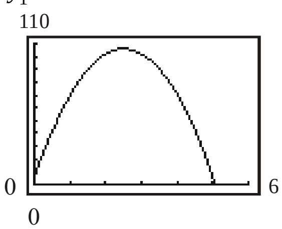
d. $y_1 = x^2 + \frac{40}{x}$



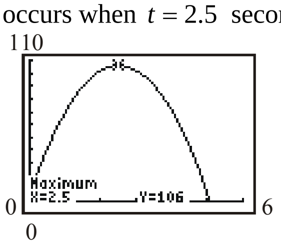


The amount of material is least when $x = 2.71$ ft.

65. a. $y_1 = -16x^2 + 80x + 6$

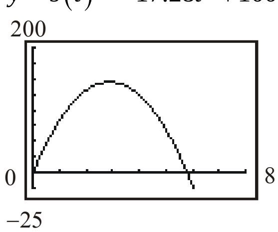


b. Use MAXIMUM. The maximum height occurs when $t = 2.5$ seconds.

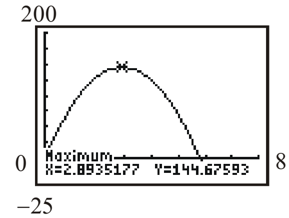


c. From the graph, the maximum height is 106 feet.

66. a. $y = s(t) = -17.28t^2 + 100t$



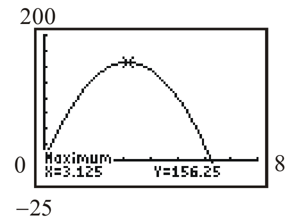
b. Use the Maximum option on the CALC menu.



The object reaches its maximum height after about 2.89 seconds.

c. From the graph in part (b), the maximum height is about 144.68 feet.

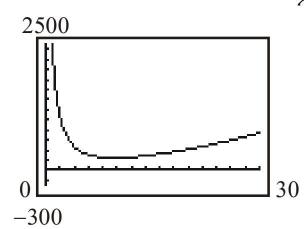
d. $s(t) = -16t^2 + 100t$



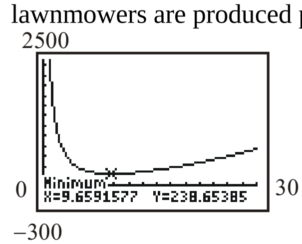
On Earth, the object would reach a maximum height of 156.25 feet after 3.125 seconds. The maximum height is slightly higher than on Saturn.

67. $\overline{C}(x) = 0.3x^2 + 21x - 251 + \frac{2500}{x}$

a. $y_1 = 0.3x^2 + 21x - 251 + \frac{2500}{x}$



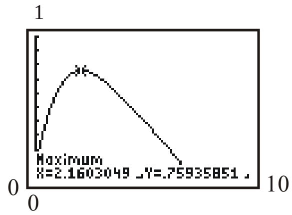
b. Use MINIMUM. The average cost is minimized when approximately 9.66 lawnmowers are produced per hour.



c. The minimum average cost is approximately \$238.65.

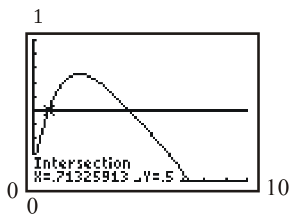
68. **a.** $C(t) = -.002t^4 + .039t^3 - .285t^2 + .766t + .085$

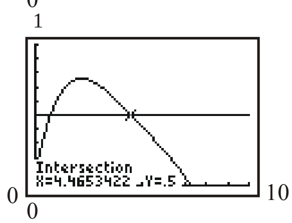
Graph the function on a graphing utility and use the Maximum option from the CALC menu.



The concentration will be highest after about 2.16 hours.

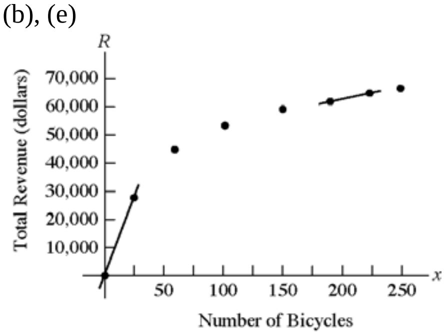
b. Enter the function in Y1 and 0.5 in Y2. Graph the two equations in the same window and use the Intersect option from the CALC menu.





After taking the medication, the woman can feed her child within the first 0.71 hours (about 42 minutes) or after 4.47 hours (about 4hours 28 minutes) have elapsed.

69. (a), (b), (e)



c. $\text{Average rate of change} = \dfrac{28000-0}{25-0} = \dfrac{28000}{25}$

$= 1120$ dollars/bicycle

d. For each additional bicycle sold between 0 and 25, the total revenue increases by (an average of) $1120.

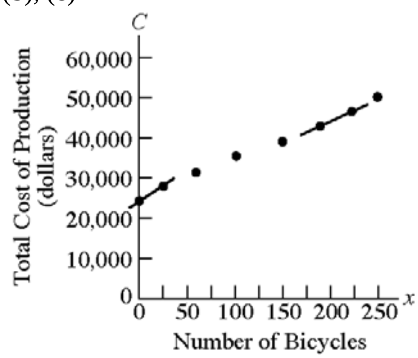
f. $\text{Average rate of change} = \dfrac{64835-62360}{223-190} = \dfrac{2475}{33}$

$= 75$ dollars per bicycle

g. For each additional bicycle sold between 190 and 223, the total revenue increases by (an average of) $75.

h. The average rate of change of revenue is decreasing as the number of bicycles increases.

70. (a), (b), (e)



c. $\text{Average rate of change} = \dfrac{27750-24000}{25-0}$

$= \dfrac{3750}{25} = 150$ dollars/bicycle

d. For each additional bicycle made between 0 and 25, the total production cost increases by (an average of) $150.

f. $\text{Average rate of change} = \dfrac{46500-42750}{223-190} = \dfrac{3750}{33}$

$= 113.64$ dollars/bicycle

g. For each additional bicycle made between 190 and 223, the total production cost increases by (an average of) $113.64.

h. The average rate of change of cost is decreasing as the number of bicycles increases.

71. $f(x) = x^2$

a. Average rate of change of *f* from $x = 0$ to $x = 1$:

$$\frac{f(1)-f(0)}{1-0} = \frac{1^2-0^2}{1} = \frac{1}{1} = 1$$

b. Average rate of change of *f* from $x=0$ to $x=0.5$:

$$\frac{f(0.5)-f(0)}{0.5-0}=\frac{(0.5)^2-0^2}{0.5}=\frac{0.25}{0.5}=0.5$$

c. Average rate of change of *f* from $x=0$ to $x=0.1$:

$$\frac{f(0.1)-f(0)}{0.1-0}=\frac{(0.1)^2-0^2}{0.1}=\frac{0.01}{0.1}=0.1$$

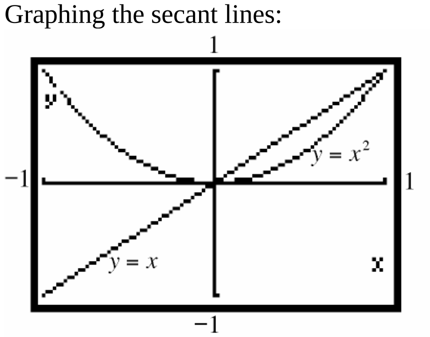
d. Average rate of change of *f* from $x=0$ to $x=0.01$:

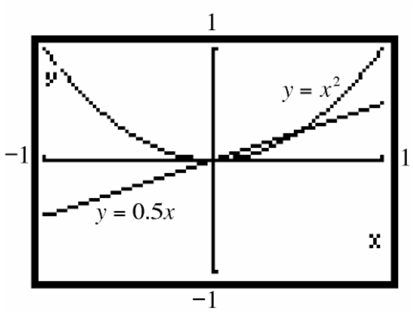
$$\frac{f(0.01)-f(0)}{0.01-0}=\frac{(0.01)^2-0^2}{0.01}$$
$$=\frac{0.0001}{0.01}=0.01$$

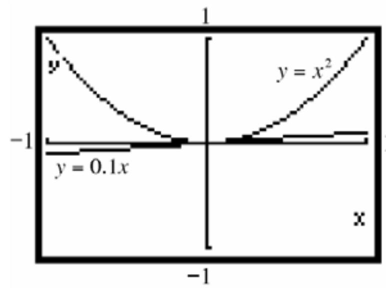
e. Average rate of change of *f* from $x=0$ to $x=0.001$:

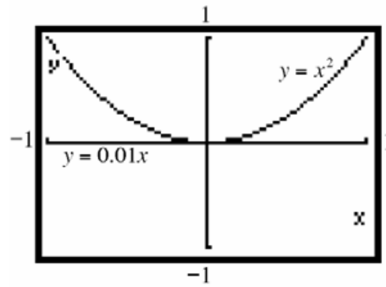
$$\frac{f(0.001)-f(0)}{0.001-0}=\frac{(0.001)^2-0^2}{0.001}$$
$$=\frac{0.000001}{0.001}=0.001$$

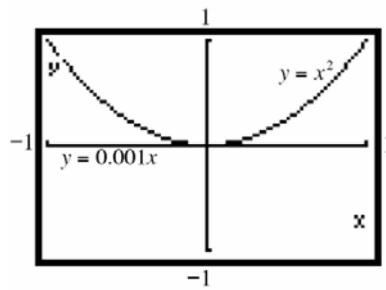
f. Graphing the secant lines:











g. The secant lines are beginning to look more and more like the tangent line to the graph of *f* at the point where x = 0.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number zero.

72. $f(x)=x^2$

a. Average rate of change of *f* from $x=1$ to $x=2$:

$$\frac{f(2)-f(1)}{2-1}=\frac{2^2-1^2}{1}=\frac{3}{1}=3$$

b. Average rate of change of *f* from $x=1$ to $x=1.5$:

$$\frac{f(1.5)-f(1)}{1.5-1}=\frac{(1.5)^2-1^2}{0.5}=\frac{1.25}{0.5}=2.5$$

c. Average rate of change of *f* from $x=1$ to $x=1.1$:

$$\frac{f(1.1)-f(1)}{1.1-1}=\frac{(1.1)^2-1^2}{0.1}=\frac{0.21}{0.1}=2.1$$

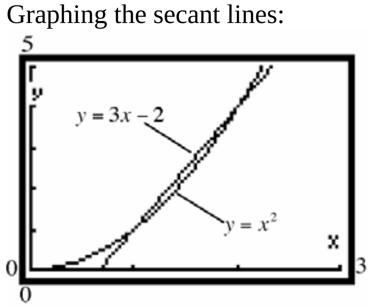
d. Average rate of change of *f* from $x=1$ to $x=1.01$:

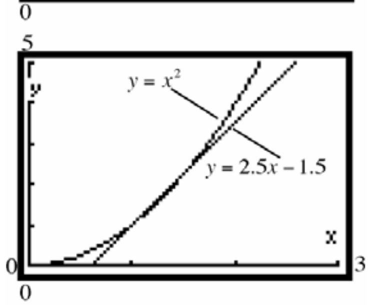
$$\frac{f(1.01)-f(1)}{1.01-1}=\frac{(1.01)^2-1^2}{0.01}$$
$$=\frac{0.0201}{0.01}=2.01$$

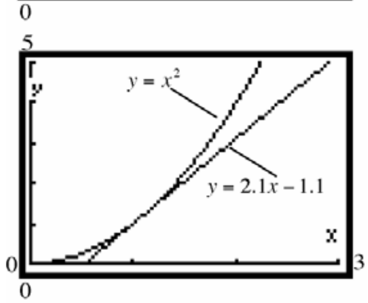
e. Average rate of change of *f* from $x=1$ to $x=1.001$:

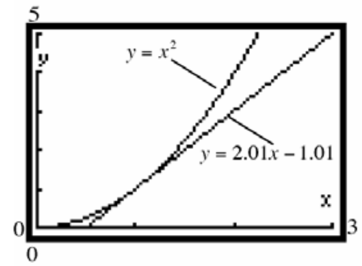
$$\frac{f(1.001)-f(1)}{1.001-1}=\frac{(1.001)^2-1^2}{0.001}$$
$$=\frac{0.002001}{0.001}=2.001$$

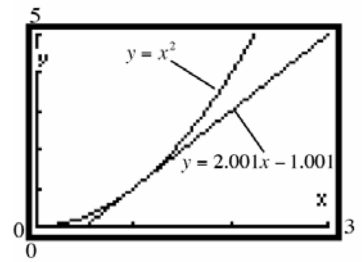
f. Graphing the secant lines:











g. The secant lines are beginning to look more and more like the tangent line to the graph of f at the point where x = 1.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number 2.

73. $f(x)=2x+5$

a. $$m_{\text{sec}}=\frac{f(x+h)-f(x)}{h}$$
$$=\frac{2(x+h)+5-2x-5}{h}$$
$$=\frac{2h}{h}=2$$

b. When $x=1$:

$h=0.5\Rightarrow m_{\text{sec}}=2$

$h=0.1\Rightarrow m_{\text{sec}}=2$

$h=0.01\Rightarrow m_{\text{sec}}=2$

as $h\to 0,\ \ m_{\text{sec}}\to 2$

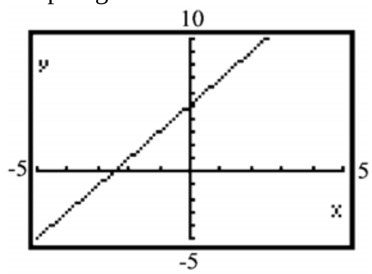
c. Using the point $(1, f(1))=(1,7)$ and slope, $m=2$, we get the secant line:

$y-7=2(x-1)$

$y-7=2x-2$

$y=2x+5$

d. Graphing:



The graph and the secant line coincide.

74. $f(x) = -3x + 2$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{-3(x+h) + 2 - (-3x+2)}{h}$

$= \dfrac{-3h}{h} = -3$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = -3$

$h = 0.1 \Rightarrow m_{\text{sec}} = -3$

$h = 0.01 \Rightarrow m_{\text{sec}} = -3$

as $h \to 0$, $m_{\text{sec}} \to -3$

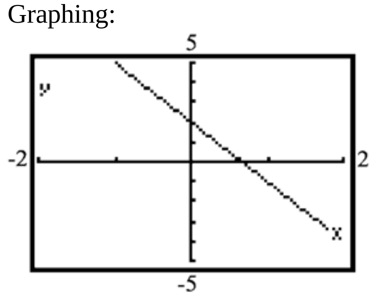
c. Using point $(1, f(1)) = (1, -1)$ and slope = -3, we get the secant line:

$y - (-1) = -3(x-1)$

$y + 1 = -3x + 3$

$y = -3x + 2$

d. Graphing:



The graph and the secant line coincide.

75. $f(x) = x^2 + 2x$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$

$= \dfrac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$

$= \dfrac{2xh + h^2 + 2h}{h}$

$= 2x + h + 2$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.5 + 2 = 4.5$

$h = 0.1 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.1 + 2 = 4.1$

$h = 0.01 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.01 + 2 = 4.01$

as $h \to 0$, $m_{\text{sec}} \to 2 \cdot 1 + 0 + 2 = 4$

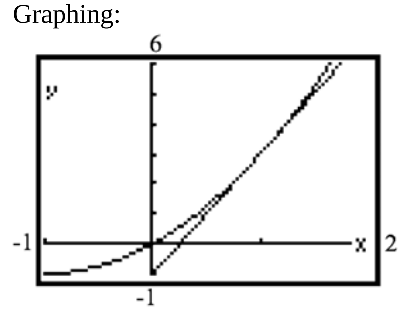
c. Using point $(1, f(1)) = (1, 3)$ and slope = 4.01, we get the secant line:

$y - 3 = 4.01(x-1)$

$y - 3 = 4.01x - 4.01$

$y = 4.01x - 1.01$

d. Graphing:



76. $f(x) = 2x^2 + x$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{2(x+h)^2 + (x+h) - (2x^2 + x)}{h}$

$= \dfrac{2(x^2 + 2xh + h^2) + x + h - 2x^2 - x}{h}$

$= \dfrac{2x^2 + 4xh + 2h^2 + x + h - 2x^2 - x}{h}$

$= \dfrac{4xh + 2h^2 + h}{h}$

$= 4x + 2h + 1$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.5) + 1 = 6$

$h = 0.1 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.1) + 1 = 5.2$

$h = 0.01 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.01) + 1 = 5.02$

as $h \to 0$, $m_{\text{sec}} \to 4 \cdot 1 + 2(0) + 1 = 5$

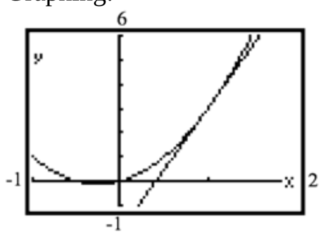
c. Using point $(1, f(1)) = (1, 3)$ and slope = 5.02, we get the secant line:

$y - 3 = 5.02(x - 1)$

$y - 3 = 5.02x - 5.02$

$y = 5.02x - 2.02$

d. Graphing:



77. $f(x) = 2x^2 - 3x + 1$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h}$

$= \dfrac{2(x^2 + 2xh + h^2) - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$

$= \dfrac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$

$= \dfrac{4xh + 2h^2 - 3h}{h}$

$= 4x + 2h - 3$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.5) - 3 = 2$

$h = 0.1 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.1) - 3 = 1.2$

$h = 0.01 \Rightarrow m_{\text{sec}} = 4 \cdot 1 + 2(0.01) - 3 = 1.02$

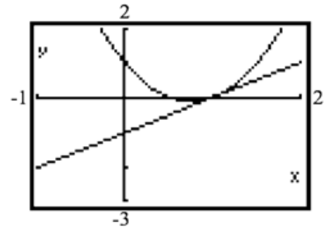
as $h \to 0$, $m_{\text{sec}} \to 4 \cdot 1 + 2(0) - 3 = 1$

c. Using point $(1, f(1)) = (1, 0)$ and slope = 1.02, we get the secant line:

$y - 0 = 1.02(x - 1)$

$y = 1.02x - 1.02$

d. Graphing:



78. $f(x) = -x^2 + 3x - 2$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{-(x+h)^2 + 3(x+h) - 2 - (-x^2 + 3x - 2)}{h}$

$= \dfrac{-(x^2 + 2xh + h^2) + 3x + 3h - 2 + x^2 - 3x + 2}{h}$

$= \dfrac{-x^2 - 2xh - h^2 + 3x + 3h - 2 + x^2 - 3x + 2}{h}$

$= \dfrac{-2xh - h^2 + 3h}{h}$

$= -2x - h + 3$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.5 + 3 = 0.5$

$h = 0.1 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.1 + 3 = 0.9$

$h = 0.01 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.01 + 3 = 0.99$

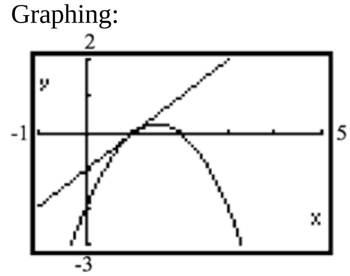
as $h \to 0$, $m_{\text{sec}} \to -2 \cdot 1 - 0 + 3 = 1$

c. Using point $(1, f(1)) = (1, 0)$ and slope = 0.99, we get the secant line:

$y - 0 = 0.99(x - 1)$

$y = 0.99x - 0.99$

d. Graphing:



79. $f(x)=\frac{1}{x}$

a. $m_{\text{sec}}=\frac{f(x+h)-f(x)}{h}=\frac{\left(\frac{1}{x+h}-\frac{1}{x}\right)}{h}$

$=\frac{\left(\frac{x-(x+h)}{(x+h)x}\right)}{h}=\left(\frac{x-x-h}{(x+h)x}\right)\left(\frac{1}{h}\right)$

$=\left(\frac{-h}{(x+h)x}\right)\left(\frac{1}{h}\right)$

$=-\frac{1}{(x+h)x}$

b. When $x = 1$,

$h=0.5 \Rightarrow m_{\text{sec}}=-\frac{1}{(1+0.5)(1)}$

$=-\frac{1}{1.5}\approx -0.667$

$h=0.1 \Rightarrow m_{\text{sec}}=-\frac{1}{(1+0.1)(1)}$

$=-\frac{1}{1.1}\approx -0.909$

$h=0.01 \Rightarrow m_{\text{sec}}=-\frac{1}{(1+0.01)(1)}$

$=-\frac{1}{1.01}\approx -0.990$

as $h\to 0$, $m_{\text{sec}}\to -\frac{1}{(1+0)(1)}=-\frac{1}{1}=-1$

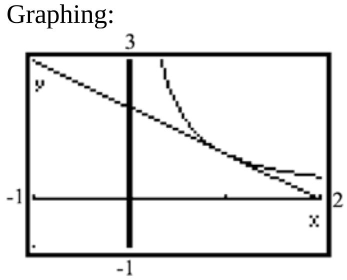
c. Using point $(1, f(1))=(1,1)$ and slope = -0.990, we get the secant line:

$y-1=-0.99(x-1)$

$y-1=-0.99x+0.99$

$y=-0.99x+1.99$

d. Graphing:



80. $f(x)=\frac{1}{x^2}$

a. $m_{\text{sec}}=\frac{f(x+h)-f(x)}{h}$

$=\frac{\left(\frac{1}{(x+h)^2}-\frac{1}{x^2}\right)}{h}$

$=\frac{\left(\frac{x^2-(x+h)^2}{(x+h)^2x^2}\right)}{h}$

$=\left(\frac{x^2-(x^2+2xh+h^2)}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)$

$=\left(\frac{-2xh-h^2}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)$

$=\frac{-2x-h}{(x+h)^2x^2}$

b. When $x = 1$,

$h=0.5 \Rightarrow m_{\text{sec}}=\frac{-2\cdot 1-0.5}{(1+0.5)^2 1^2}\approx -1.111$

$h=0.1 \Rightarrow m_{\text{sec}}=\frac{-2\cdot 1-0.1}{(1+0.1)^2 1^2}\approx -1.736$

$h=0.01 \Rightarrow m_{\text{sec}}=\frac{-2\cdot 1-0.01}{(1+0.01)^2 1^2}\approx -1.970$

as $h\to 0$, $m_{\text{sec}}\to \frac{-2\cdot 1-0}{(1+0)^2 1^2}=-2$

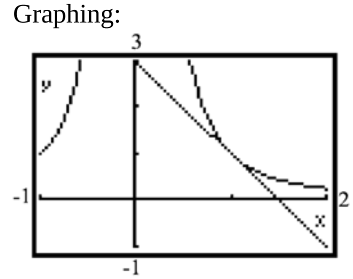
c. Using point $(1, f(1))=(1,1)$ and slope = -1.970, we get the secant line:

$y-1=-1.970(x-1)$

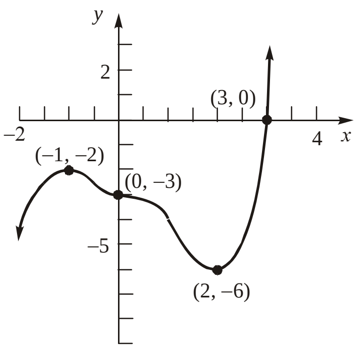
$y-1=-1.97x+1.97$

$y=-1.97x+2.97$

d. Graphing:



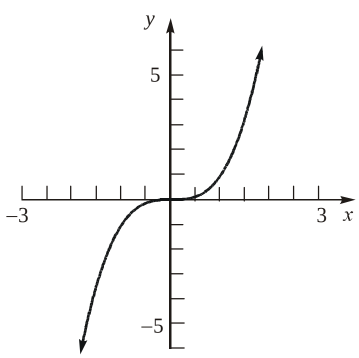
81. Answers will vary. One possibility follows:



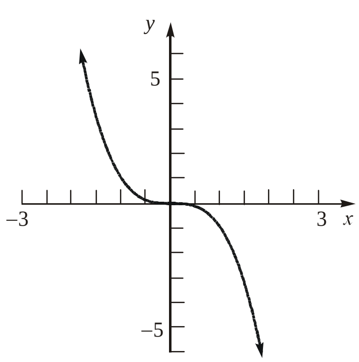
82. Answers will vary. See solution to Problem 81 for one possibility.

83. A function that is increasing on an interval can have at most one x-intercept on the interval. The graph of f could not "turn" and cross it again or it would start to decrease.

84. An increasing function is a function whose graph goes up as you read from left to right.

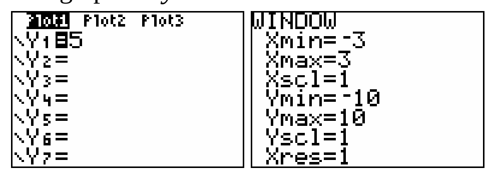


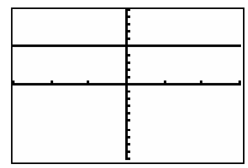
A decreasing function is a function whose graph goes down as you read from left to right.



85. To be an even function we need $f(-x) = f(x)$ and to be an odd function we need $f(-x) = -f(x)$. In order for a function be both even and odd, we would need $f(x) = -f(x)$. This is only possible if $f(x) = 0$.

86. The graph of $y = 5$ is a horizontal line.

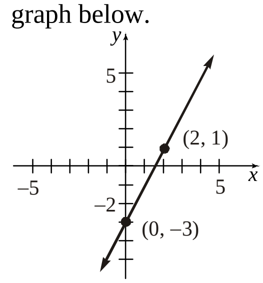




The local maximum is $y = 5$ and it occurs at each *x*-value in the interval.

Section 2.4

1. From the equation $y = 2x - 3$, we see that the y-intercept is -3. Thus, the point $(0, -3)$ is on the graph. We can obtain a second point by choosing a value for *x* and finding the corresponding value for *y*.
Let $x = 2$, then $y = 2(2) - 3 = 1$. Thus, the point $(2,1)$ is also on the graph. Plotting the two points and connecting with a line yields the graph below.



2. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-5}{-1-2} = \dfrac{-2}{-3} = \dfrac{2}{3}$

3. We can use the point-slope form of a line to obtain the equation.

$$y - y_1 = m(x - x_1)$$
$$y - 5 = -3(x - (-1))$$
$$y - 5 = -3(x + 1)$$
$$y - 5 = -3x - 3$$
$$y = -3x + 2$$

4.
$$6x - 900 = -15x + 2850$$
$$21x - 900 = 2850$$
$$21x = 3750$$
$$x = \frac{1250}{7}$$

5. slope; y-intercept

6. scatter diagram

7. $y = kx$

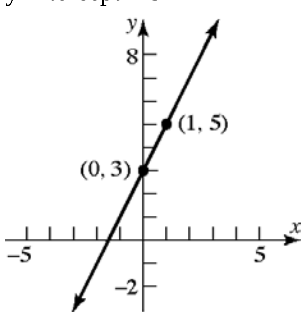
8. True

9. True

10. True

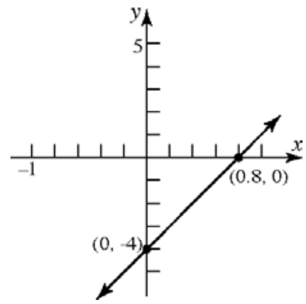
11. $f(x) = 2x + 3$

Slope = average rate of change = 2;
y-intercept = 3



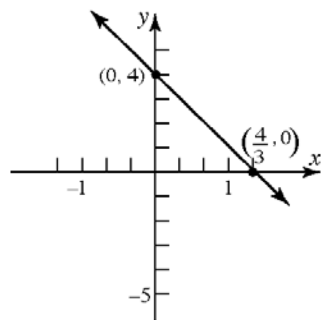
12. $g(x) = 5x - 4$

Slope = average rate of change = 5;
y-intercept = –4



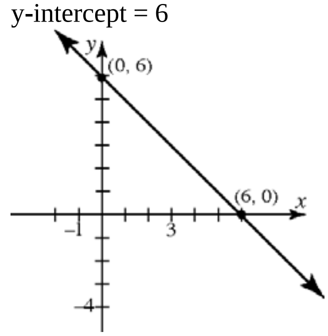
13. $h(x) = -3x + 4$

Slope = average rate of change = –3;
y-intercept = 4



14. $p(x) = -x + 6$

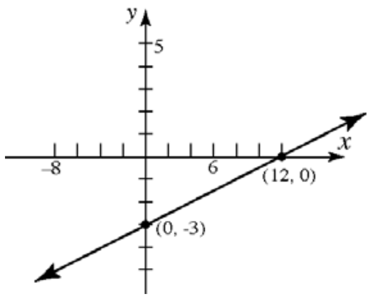
Slope = average rate of change = –1;
y-intercept = 6



15. $f(x) = \frac{1}{4}x - 3$

Slope = average rate of change = $\frac{1}{4}$;

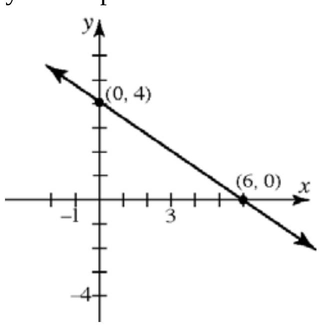
y-intercept = –3



16. $h(x) = -\frac{2}{3}x + 4$

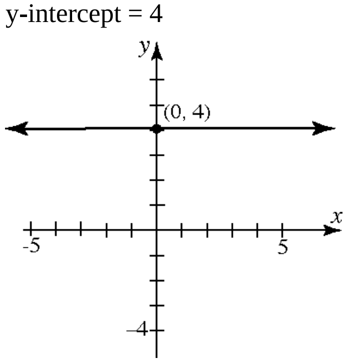
Slope = average rate of change = $-\frac{2}{3}$;

y-intercept = 4



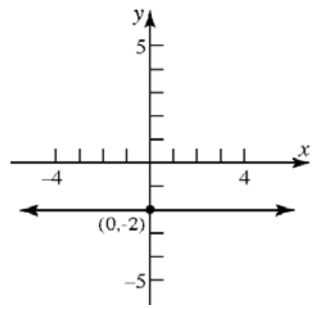
17. $F(x) = 4$

Slope = average rate of change = 0;
y-intercept = 4



18. $G(x) = -2$

Slope = average rate of change = 0;
y-intercept = –2



19. Linear, $m > 0$

20. Nonlinear

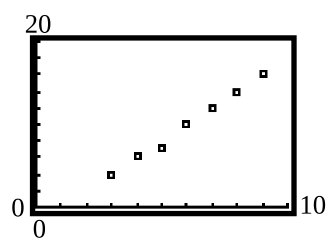
21. Linear, $m < 0$

22. No relation

23. Nonlinear

24. Nonlinear

25. **a.**



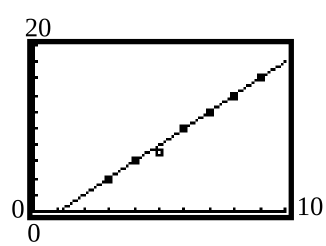
b. Answers will vary. We select (3, 4) and (9, 16). The slope of the line containing these points is:

$$m = \frac{16-4}{9-3} = \frac{12}{6} = 2$$

The equation of the line is:

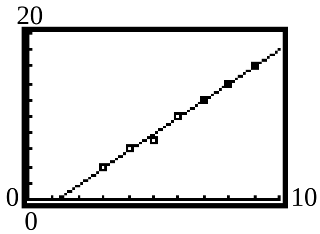
$$y - y_1 = m(x - x_1)$$
$$y - 4 = 2(x - 3)$$
$$y - 4 = 2x - 6$$
$$y = 2x - 2$$

c.

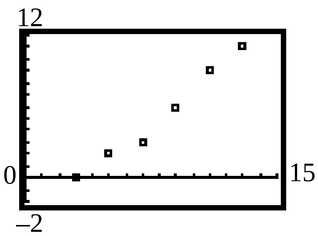


d. Using the LINear REGresssion program, the line of best fit is:
$y = 2.0357x - 2.3571$

e.



26. a.



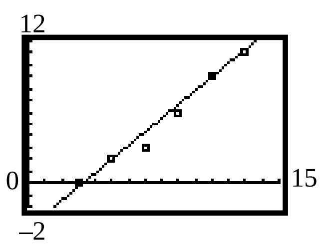
b. Selection of points will vary. We select (3, 0) and (13, 11). The slope of the line containing these points is:

$$m = \frac{11-0}{13-3} = \frac{11}{10} = 1.1$$

The equation of the line is:

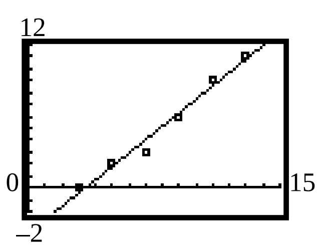
$$y - y_1 = m(x - x_1)$$
$$y - 0 = 1.1(x - 3)$$
$$y - 0 = 1.1x - 3.3$$
$$y = 1.1x - 3.3$$

c.

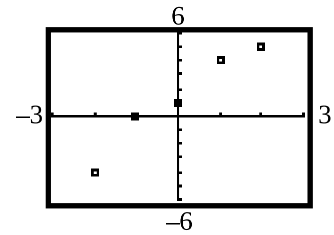


d. Using the LINear REGression program, the line of best fit is:
$y = 1.1286x - 3.8619$

e.



27. a.



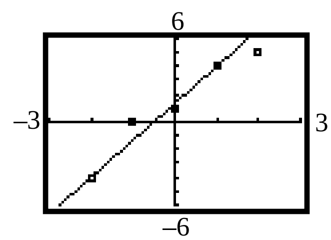
b. Answers will vary. We select (–2,–4) and (1, 4). The slope of the line containing these points is:

$$m = \frac{4-(-4)}{1-(-2)} = \frac{8}{3}$$

The equation of the line is:

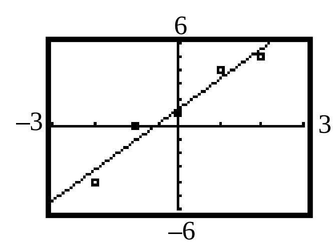
$$y - y_1 = m(x - x_1)$$
$$y - (-4) = \frac{8}{3}(x - (-2))$$
$$y + 4 = \frac{8}{3}x + \frac{16}{3}$$
$$y = \frac{8}{3}x + \frac{4}{3}$$

c.

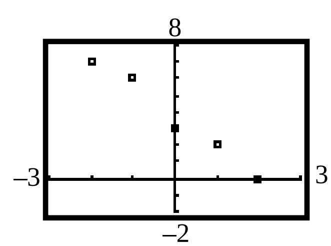


d. Using the LINear REGresssion program, the line of best fit is:
$y = 2.2x + 1.2$

e.



28. a.



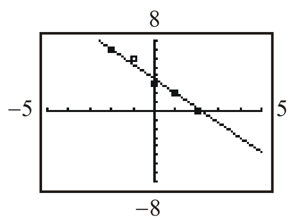
b. Selection of points will vary. We select (–2, 7) and (1, 2). The slope of the line containing these points is:

$$m = \frac{2-7}{1-(-2)} = \frac{-5}{3} = -\frac{5}{3}$$

The equation of the line is:

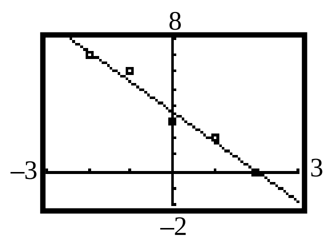
$$y - y_1 = m(x - x_1)$$
$$y - 7 = -\frac{5}{3}(x - (-2))$$
$$y - 7 = -\frac{5}{3}x - \frac{10}{3}$$
$$y = -\frac{5}{3}x + \frac{11}{3}$$

c.

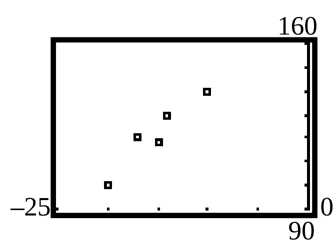


d. Using the LINear REGression program, the line of best fit is:
$y = -1.8x + 3.6$

e.



29. a.



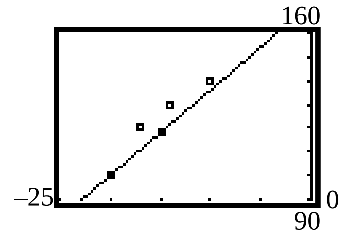
b. Answers will vary. We select (–20,100) and (–15,118). The slope of the line containing these points is:

$$m = \frac{118-100}{-15-(-20)} = \frac{18}{5} = 3.6$$

The equation of the line is:

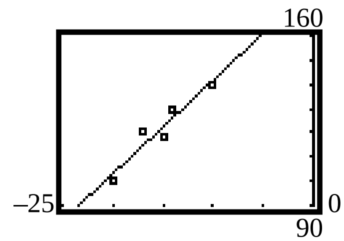
$$y - y_1 = m(x - x_1)$$
$$y - 100 = 3.6(x - (-20))$$
$$y - 100 = 3.6x + 72$$
$$y = 3.6x + 172$$

c.

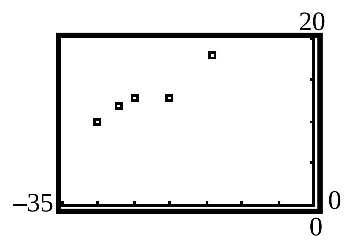


d. Using the LINear REGresssion program, the line of best fit is:
$y = 3.8613x + 180.2920$

e.



30. a.



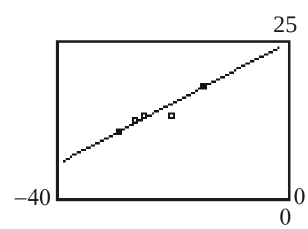
b. Selection of points will vary. We select (–30, 10) and (–14, 18). The slope of the line containing these points is:

$$m = \frac{18-10}{-14-(-30)} = \frac{8}{16} = 0.5$$

The equation of the line is:

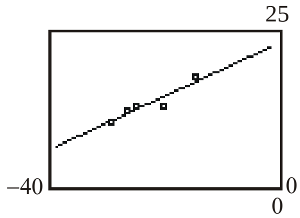
$$y - y_1 = m(x - x_1)$$
$$y - 10 = 0.5(x - (-30))$$
$$y - 10 = 0.5x + 15$$
$$y = 0.5x + 25$$

c.



d. Using the LINear REGression program, the line of best fit is:
$y = 0.4421x + 23.4559$

e.



31. a. $C(x) = 0.25x + 35$

$C(40) = 0.25(40) + 35 = 45$

The moving truck will cost $45.00 if you drive 40 miles.

b. $80 = 0.25x + 35$

$45 = 0.25x$

$180 = x$

If the cost of the truck is $80.00, you drove for 180 miles.

c. $100 = 0.25x + 35$

$65 = 0.25x$

$260 = x$

To keep the cost below $100, you must drive less than 260 miles.

32. a. $C(x) = 0.38x + 5$

$C(50) = 0.38(50) + 5 = 24$

If you talk for 50 minutes, the cost will be $24.00.

b. $29.32 = 0.38x + 5$

$24.32 = 0.38x$

$64 = x$

If the monthly bill is $29.32, you would have used the phone for 64 minutes.

c. $60 = 0.38x + 5$

$55 = 0.38x$

$144.74 \approx x$

To stay within budget, you can talk for no more than 144 minutes.

33. a. $B(t) = 19.25t + 585.72$

$B(10) = 19.25(10) + 585.72 = 778.22$

The average monthly benefit in 2000 was $778.22.

b. $893.72 = 19.25t + 585.72$

$308 = 19.25t$

$16 = t$

The average monthly benefit will be $893.72 in 2006.

c. $1000 = 19.25t + 585.72$

$414.28 = 19.25t$

$21.52 \approx t$

The average monthly benefit will exceed $1000 in 2012.

34. a. $H(t) = 26t + 411$

$H(10) = 26(10) + 411 = 671$

The total private health expenditures in 2000 was $671 billion.

b. $879 = 26t + 411$

$468 = 26t$

$18 = t$

Total private health expenditures will be $879 billion in 2008.

c. $1000 = 26t + 411$

$589 = 26t$

$22.65 = t$

Total private health expenditures will exceed $1 trillion in 2013.

35. a. $S(p) = D(p)$

$-200 + 50p = 1000 - 25p$

$75p = 1200$

$p = 16$

The equilibrium price is $16.

$S(16) = -200 + 50(16) = 600$

The equilibrium quantity is 600 T-shirts.

b. $D(p) > S(p)$

$1000 - 25p > -200 + 50p$

$-75p > -1200$

$p < 16$

The quantity demanded will exceed the quantity supplied if $0 < p < \$16$.

c. If demand is higher than supply, generally the price will increase. The price will continue to increase towards the equilibrium point.

36. a.
$$S(p) = D(p)$$
$$-2000 + 3000p = 10,000 - 1000p$$
$$4000p = 12,000$$
$$p = 3$$
The equilibrium price is \$3.
$S(3) = -2000 + 3000(3) = 7000$
The equilibrium quantity is 7000 hot dogs.

b.
$$D(p) < S(p)$$
$$10,000 - 1000p < -2000 + 3000p$$
$$-4000p < -12,000$$
$$p > 3$$
The quantity demanded will be less than the quantity supplied if $p > \$3$.

c. If demand is less than the quantity supplied, the price will generally decrease. The price will continue to decrease towards the equilibrium point.

37. a.
$$R(x) = C(x)$$
$$8x = 4.5x + 17,500$$
$$3.5x = 17,500$$
$$x = 5000$$
The company must sell 5000 units to break even.

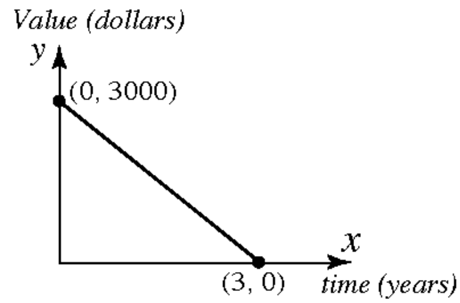
b. To make a profit, the company must sell more than 5000 units.

38. a.
$$R(x) = C(x)$$
$$12x = 10x + 15,000$$
$$2x = 15,000$$
$$x = 7500$$
The company must sell 7500 units to break even.

b. To make a profit, the company must sell more than 7500 units.

39. a. Consider the data points (x, y), where x = the age in years of the computer and y = the value in dollars of the computer. So we have the points $(0,3000)$ and $(3,0)$. The slope formula yields:
$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{0-3000}{3-0} = \frac{-3000}{3} = -1000 = m$$
$(0,3000)$ is the y-intercept, so $b = 3000$
Therefore, the linear function is $V(x) = mx + b = -1000x + 3000$.

b. The graph of $V(x) = -1000x + 3000$



c. The computer's value after 2 years is given by
$$V(2) = -1000(2) + 3000$$
$$= -2000 + 3000 = \$1000$$

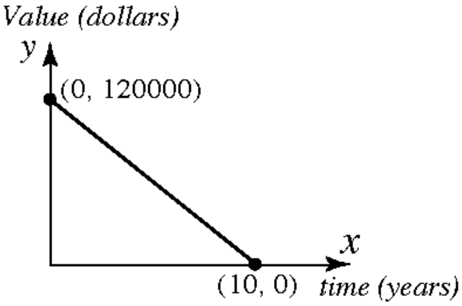
d. To find when the computer will be worth \$2000, we solve the following:
$$2000 = -1000x + 3000$$
$$-1000 = -1000x$$
$$1 = x$$
The computer will be worth \$2000 after 1 year.

40. a. Consider the data points (x, y), where x = the age in years of the machine and y = the value in dollars of the machine. So we have the points $(0,120000)$ and $(10,0)$. The slope formula yields:

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{0-120000}{10-0} = \frac{-120000}{10}$$
$$= -12000 = m$$

$(0,120000)$ is the y-intercept, so $b = 120000$. Therefore, the linear function is $V(x) = mx + b = -12000x + 120000$.

b. The graph of $V(x) = -12000x + 120000$



c. The machine's value after 4 years is given by

$$V(4) = -12000(4) + 120000$$
$$= -48000 + 120000$$
$$= \$72000$$

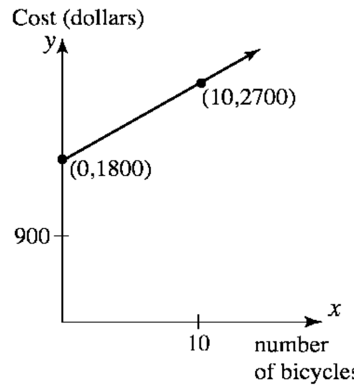
d. To find when the machine will be worth \$60,000, we solve the following:

$$60,000 = -12,000x + 120,000$$
$$-60,000 = -12,000x$$
$$5 = x$$

The machine will be worth \$60,000 after 5 years.

41. a. Let x = the number of bicycles manufactured. We can use the cost function $C(x) = mx + b$, with m = 90 and b = 1800. Therefore $C(x) = 90x + 1800$

b. The graph of $C(x) = 90x + 1800$



c. The cost of manufacturing 14 bicycles is given by $C(14) = 90(14) + 1800 = \$3060$.

d. To determine the number of bicycles, we solve the following:

$$3780 = 90x + 1800$$
$$1980 = 90x$$
$$22 = x$$

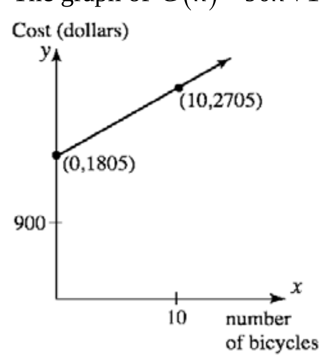
The company can manufacture 22 bicycles for \$3780.

42. a. The new daily fixed cost is

$$1800 + \frac{100}{20} = \$1805.$$

b. Let x = the number of bicycles manufactured. We can use the cost function $C(x) = mx + b$, with m = 90 and b = 1805. Therefore $C(x) = 90x + 1805$.

c. The graph of $C(x) = 90x + 1805$



d. The cost of manufacturing 14 bicycles is given by $C(14) = 90(14) + 1805 = \$3065$.

e. To determine the number of bicycles, we solve the following:
$3780 = 90x + 1805$
$1975 = 90x$
$21.94 \approx x$
The company can manufacture 21 complete bicycles for $3780.

43. a. Let x = number of miles driven and C = cost in dollars to rent a truck for one day. Since the fixed daily charge is $29 and the variable mileage charge is $0.07 per mile, we have $C(x) = 0.07x + 29$.

b. $C(110) = 0.07(110) + 29 = 36.70$
$C(230) = 0.07(230) + 29 = 45.10$
It will cost $36.70 for one day if the truck is driven 110 miles, and it will cost $45.10 if the truck is driven 230 miles.

44. a. Let x = number of minutes talking and C = cost in dollars. Since the fixed charge is $5 and the variable time charge is $0.05 per minute, we have
$C(x) = 0.05x + 5$.

b. $C(105) = 0.05(105) + 5 = \$10.25$
$C(180) = 0.05(180) + 5 = \$14.00$
The plan will cost $10.25 if you talk for 105 minutes, and it will cost $14.00 if you talk for 180 minutes.

45. Let $p =$ the monthly payment and B = the amount borrowed. Consider the ordered pair (B, p). We can use the points $(0,0)$ and $(1000, 6.49)$.
Now compute the slope:
$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{6.49 - 0}{1000 - 0} = \frac{6.49}{1000} = 0.00649$$
Therefore we have the linear function
$p(B) = 0.00649B + 0 = 0.00649B$.
If $B = 145000$, then
$p = (0.00649)(145000) = \$941.05$.

46. Let $p =$ the monthly payment and B = the amount borrowed. Consider the ordered pair (B, p). We can use the points $(0,0)$ and $(1000, 8.99)$. Now compute the slope:
$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{8.99 - 0}{1000 - 0} = \frac{8.99}{1000} = 0.00899$$
Therefore we have the linear function
$p(B) = 0.00899B + 0 = 0.00899B$.
If $B = 175000$, then
$p = (0.00899)(175000) = \$1573.25$.

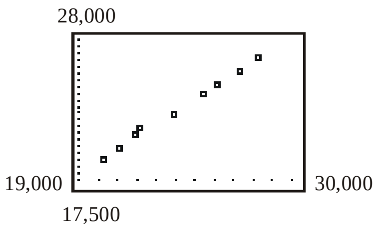
47. Let R = the revenue and g = the number of gallons of gasoline sold. Consider the ordered pair (g, R). We can use the points $(0,0)$ and $(12, 23.40)$. Now compute the slope:
$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{23.40 - 0}{12 - 0} = \frac{23.40}{12} = 1.95$$
Therefore we have the linear function
$R(g) = 1.95g + 0 = 1.95g$.
If $g = 10.5$, then $R = (1.95)(10.5) = \$20.48$.

48. Let C = the cost and A = the number of gallons almonds purchased. Consider the ordered pair (A, C). We can use the points $(0,0)$ and $(5, 23.75)$. Now compute the slope:
$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{23.75 - 0}{5 - 0} = \frac{23.75}{5} = 4.75$$
Therefore we have the linear function
$C(A) = 4.75A + 0 = 4.75A$.
If $A = 3.5$, then $C = (4.75)(3.5) = \$16.63$.

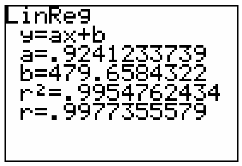
49. $W = kS$
$1.875 = k(15)$
$0.125 = k$
For 40 gallons of sand:
$W = 0.125(40) = 5$ gallons of water.

50. $v = kt$
$64 = k(2) \Rightarrow k = 32$
in 3 seconds
$v = (32)(3) = 96$ feet per second

51. a.



b.



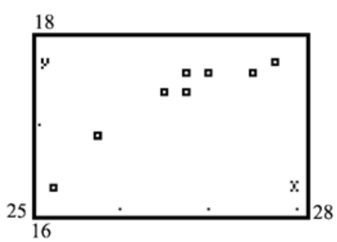
$C(I) = 0.9241I + 479.6584$

c. The slope indicates that for each \$1 increase in per capita disposable income, there is an increase of \$0.92 in per capita consumption.

d. $C(28,750) = 0.9241(28,750) + 479.6584$
$= 27,047.53$
When disposable income is \$28,750, the per capita consumption is about \$27,048.

e. $26,900 = 0.9241I + 479.6584$
$26,420.3416 = 0.9241I$
$28,590.35 = I$
When comsumption is \$26,500, the per capita disposable income is about \$28,590.

52. a.



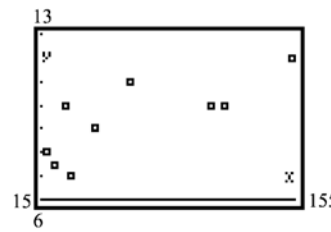
b. Using the LINear REGression program, the line of best fit is:
$C(H) = 0.3734H + 7.3268$

c. For each 1 inch increase in height, the circumference increases by 0.3734 inch.

d. $C(26) = 0.3734(26) + 7.3268 \approx 17.035$ inches

e. To find the height, we solve the following equation:
$17.4 = 0.3734H + 7.3268$
$10.0732 = 0.3734H$
$26.98 \approx H$
A child with a head circumference of 17.4 inches would have a height of about 26.98 inches.

53. a.

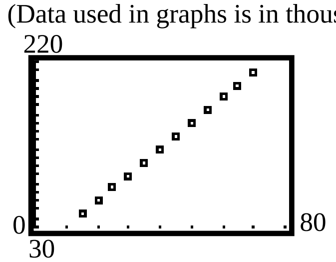


b. Using the LINear REGression program, the line of best fit is:
$L(G) = 0.0261G + 7.8738$

c. For each 1 day increase in Gestation period, the life expectancy increases by 0.0261 years (about 9.5 days).

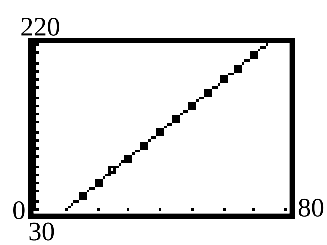
d. $L(89) = 0.0261(89) + 7.8738 \approx 10.2$ years

54. a. (Data used in graphs is in thousands.)



b. $L = 2.9814I - 0.0761072$ (data in thousands) or $L = 2.9814I - 76.1072$

c.

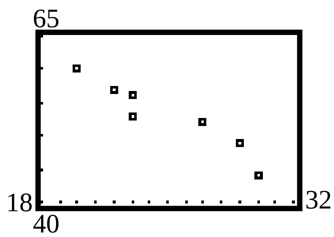


d. As annual income increases by \$1, the loan amount increases by \$2.98.

e. $L = 2.9814(42) - 0.0761072 = 125.143$
A person with an annual income of \$42,000 would qualify for a loan of about \$125,143.

55. a. The relation is not a function because 23 is paired with both 56 and 53.

b.



c. Using the LINear REGression program, the line of best fit is:
$D = -1.3355p + 86.1974$

d. As the price of the jeans increases by \$1, the demand for the jeans decreases by 1.3355 pairs per day.

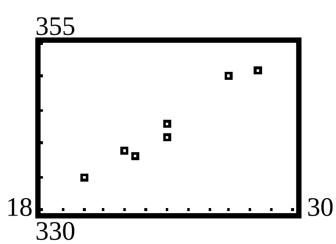
e. $D(p) = -1.3355p + 86.1974$

f. Domain: $\{p \mid 0 < p < 64\}$
Note that the *p*-intercept is roughly 64.54 and that the number of pairs cannot be negative.

g. $D(28) = -1.3355(28) + 86.1974$
≈ 48.8034
Demand is about 49 pairs.

56. a. The relation is not a function because 24 is paired with both 343 and 341.

b.



c. Using the LINear REGression program, the line of best fit is:
$S = 2.0667A + 292.8869$

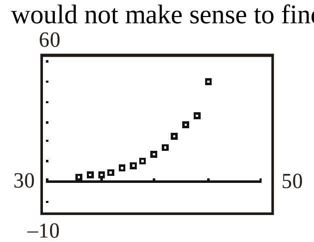
d. As the advertising expenditure increases by \$1000, the sales increase by \$2066.70.

e. $S(A) = 2.0667A + 292.8869$

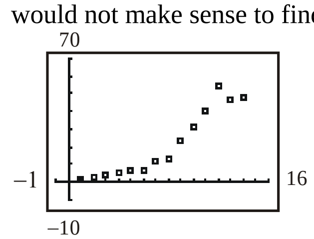
f. Domain: $\{A \mid A \geq 0\}$

g. $S(25) = 2.0667(25) + 292.8869$
≈ 344.5544
Sales are about \$344,554.

57. The data do not follow a linear pattern so it would not make sense to find the line of best fit.

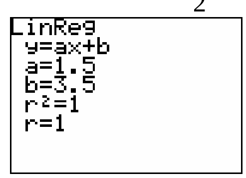


58. The data do not follow a linear pattern so it would not make sense to find the line of best fit.



59. A linear function is odd if the y-intercept is 0. That is, if the line passes through the origin. A linear function can be even if the slope is 0.

60. Using the ordered pairs $(1,5)$ and $(3,8)$, the line of best fit is $y = \frac{3}{2}x + \frac{7}{2}$.



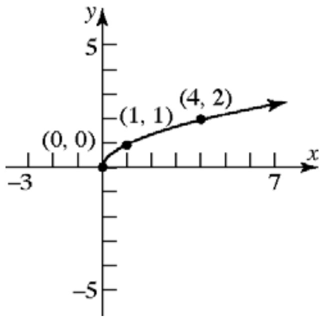
The correlation coefficient is $r = 1$. This makes sense because two points completely determine a line.

61. A correlation coefficient of 0 implies that there is no linear relationship between the data.

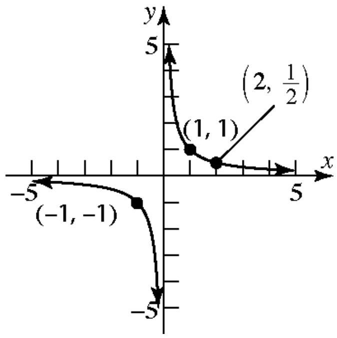
62. If the student's average in the class is directly proportional to the amount of time the student studies, the relationship between the average and time studying would be linear. That is, we could express the relationship as $A = kT$ where T is the time spent studying and A is the student's average.

Section 2.5

1. $y = \sqrt{x}$



2. $y = \dfrac{1}{x}$



3. $y = x^3 - 8$

y-intercept:
Let $x = 0$, then $y = (0)^3 - 8 = -8$.

x-intercept:
Let $y = 0$, then

$$0 = x^3 - 8$$
$$x^3 = 8$$
$$x = 2$$

The intercepts are $(0, -8)$ and $(2, 0)$.

4. less

5. piecewise defined

6. True

7. False; the cube root function is odd and increasing on the interval $(-\infty, \infty)$.

8. False; the domain and range of the reciprocal function are both the set of real numbers except for 0.

9. C

10. A

11. E

12. G

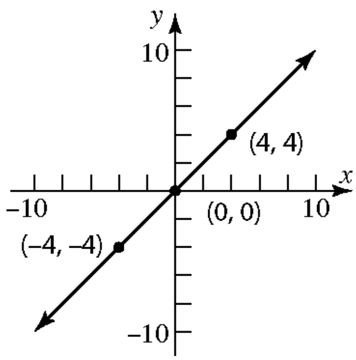
13. B

14. D

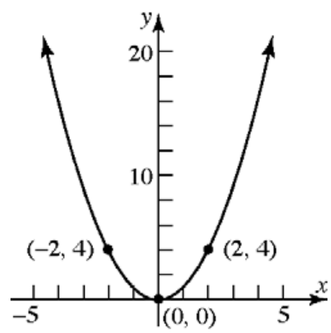
15. F

16. H

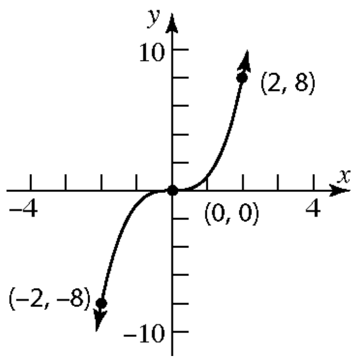
17. $f(x) = x$



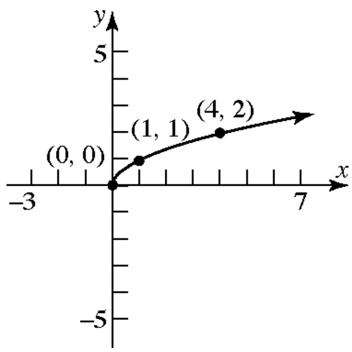
18. $f(x) = x^2$



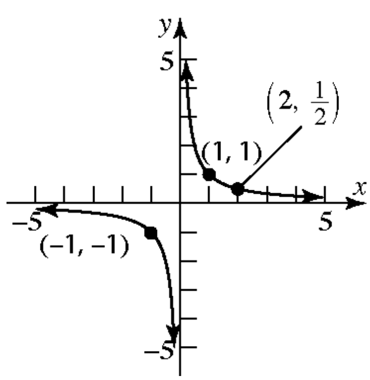
19. $f(x) = x^3$



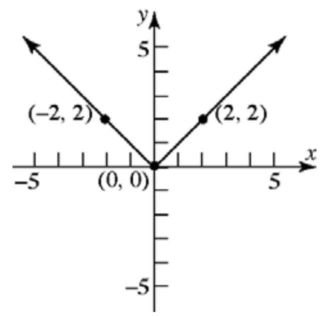
20. $f(x) = \sqrt{x}$



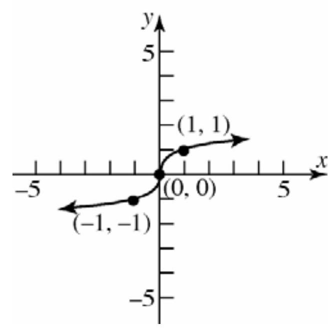
21. $f(x) = \dfrac{1}{x}$



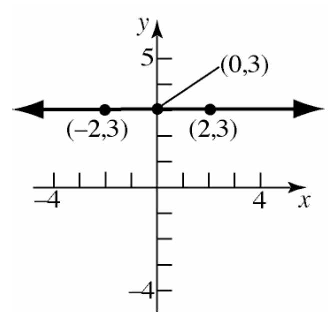
22. $f(x) = |x|$



23. $f(x) = \sqrt[3]{x}$



24. $f(x) = 3$



25. **a.** $f(-2) = (-2)^2 = 4$

b. $f(0) = 2$

c. $f(2) = 2(2) + 1 = 5$

26. **a.** $f(-1) = (-1)^3 = -1$

b. $f(0) = 3(0) + 2 = 2$

c. $f(1) = 3(1) + 2 = 5$

27. **a.** $f(1.2) = \text{int}(2(1.2)) = \text{int}(2.4) = 2$

b. $f(1.6) = \text{int}(2(1.6)) = \text{int}(3.2) = 3$

c. $f(-1.8) = \text{int}(2(-1.8)) = \text{int}(-3.6) = -4$

28. **a.** $f(1.2) = \text{int}\left(\frac{1.2}{2}\right) = \text{int}(0.6) = 0$

b. $f(1.6) = \text{int}\left(\frac{1.6}{2}\right) = \text{int}(0.8) = 0$

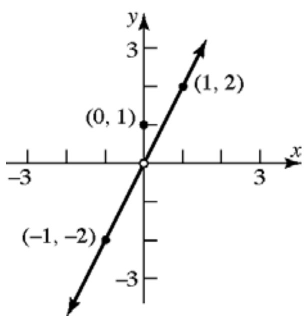
c. $f(-1.8) = \text{int}\left(\frac{-1.8}{2}\right) = \text{int}(-0.9) = -1$

29. $f(x) = \begin{cases} 2x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept: (0,1)

c. Graph:



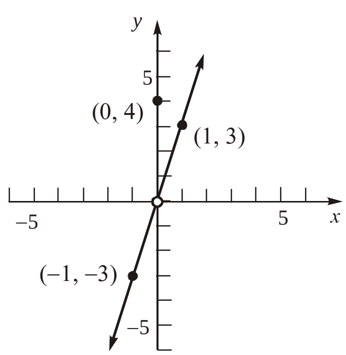
d. Range: $\{y \mid y \neq 0\}$

30. $f(x) = \begin{cases} 3x & \text{if } x \neq 0 \\ 4 & \text{if } x = 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept: (0, 4)

c. Graph:



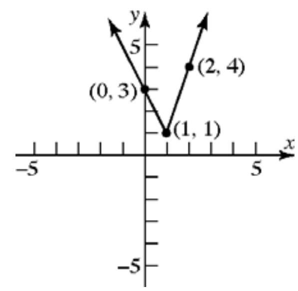
d. Range: $\{y \mid y \neq 0\}$

31. $f(x) = \begin{cases} -2x + 3 & \text{if } x < 1 \\ 3x - 2 & \text{if } x \geq 1 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept: (0,3)

c. Graph:



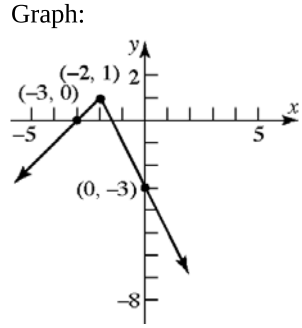
d. Range: $\{y \mid y \geq 1\}$

32. $f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ -2x - 3 & \text{if } x \geq -2 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercepts: (–3, 0), (–1.5, 0)
y-intercept: (0, –3)

c. Graph:



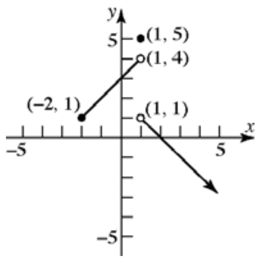
d. Range: $\{y \mid y \leq 1\}$

33. $f(x) = \begin{cases} x + 3 & \text{if } -2 \leq x < 1 \\ 5 & \text{if } x = 1 \\ -x + 2 & \text{if } x > 1 \end{cases}$

a. Domain: $\{x \mid x \geq -2\}$

b. x-intercept: (2, 0)
y-intercept: (0, 3)

c. Graph:



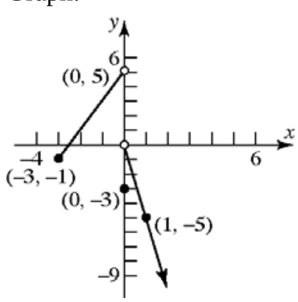
d. Range: $\{y \mid y < 4 \text{ and } y = 5\}$

34. $f(x) = \begin{cases} 2x+5 & \text{if } -3 \le x < 0 \\ -3 & \text{if } x = 0 \\ -5x & \text{if } x > 0 \end{cases}$

a. Domain: $\{x \mid x \ge -3\}$

b. x-intercept: (–2.5, 0)
y-intercept: (0, –3)

c. Graph:



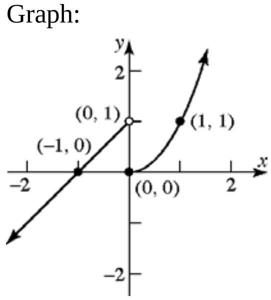
d. Range: $\{y \mid y < 5\}$

35. $f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercepts: (–1,0), (0,0)
y-intercept: (0,0)

c. Graph:



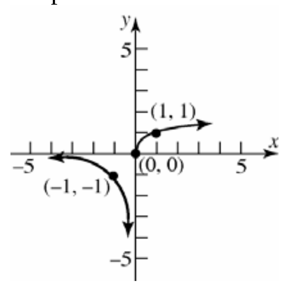
d. Range: $\{y \mid y \text{ is any real number}\}$

36. $f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ \sqrt[3]{x} & \text{if } x \ge 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: (0,0)
y-intercept: (0,0)

c. Graph:



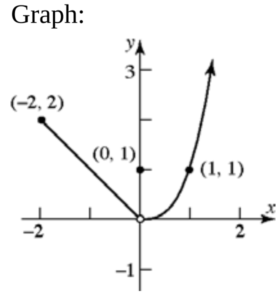
d. Range: $\{y \mid y \text{ is any real number}\}$

37. $f(x) = \begin{cases} |x| & \text{if } -2 \le x < 0 \\ 1 & \text{if } x = 0 \\ x^3 & \text{if } x > 0 \end{cases}$

a. Domain: $\{x \mid x \ge -2\}$

b. x-intercept: none
y-intercept: (0, 1)

c. Graph:



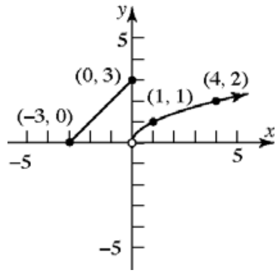
d. Range: $\{y \mid y > 0\}$

38. $f(x) = \begin{cases} 3+x & \text{if } -3 \le x < 0 \\ 3 & \text{if } x = 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$

a. Domain: $\{x \mid x \ge -3\}$

b. x-intercept: (–3, 0)
y-intercept: (0, 3)

c. Graph:



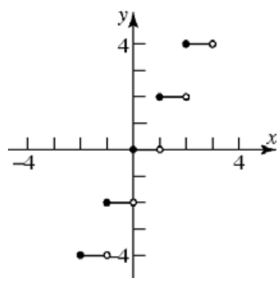
d. Range: $\{y \mid y \ge 0\}$

39. $f(x) = 2\,\text{int}(x)$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercepts: all ordered pairs $(x, 0)$ when $0 \le x < 1$.
y-intercept: $(0,0)$

c. Graph:



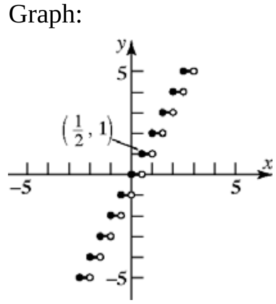
d. Range: $\{y \mid y \text{ is an even integer}\}$

40. $f(x) = \text{int}(2x)$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercepts: all ordered pairs $(x, 0)$ when $0 \le x < \dfrac{1}{2}$.
y-intercept: $(0,0)$

c. Graph:



d. Range: $\{y \mid y \text{ is an integer}\}$

41. $f(x) = \begin{cases} -x & \text{if } -1 \le x \le 0 \\ \dfrac{1}{2}x & \text{if } 0 < x \le 2 \end{cases}$

42. $f(x) = \begin{cases} x & \text{if } -1 \le x \le 0 \\ 1 & \text{if } 0 < x \le 2 \end{cases}$

43. $f(x) = \begin{cases} -x & \text{if } x \le 0 \\ -x+2 & \text{if } 0 < x \le 2 \end{cases}$

44. $f(x) = \begin{cases} 2x+2 & \text{if } -1 \le x \le 0 \\ x & \text{if } x > 0 \end{cases}$

45. $C = \begin{cases} 35 & \text{if } 0 < x \le 300 \\ 0.40x - 85 & \text{if } x > 300 \end{cases}$

a. $C(200) = \$35.00$

b. $C(365) = 0.40(365) - 85 = \$61.00$

c. $C(301) = 0.40(301) - 85 = \$35.40$

46. $F(x) = \begin{cases} 3 & \text{if } 0 < x \le 3 \\ 5\,\text{int}(x+1)+1 & \text{if } 3 < x < 9 \\ 50 & \text{if } 9 \le x \le 24 \end{cases}$

a. $F(2) = 3$
Parking for 2 hours costs \$3.

b. $F(7) = 5\,\text{int}(7+1)+1 = 41$
Parking for 7 hours costs \$41.

c. $F(15) = 50$
Parking for 15 hours costs \$50.

d. $24 \text{ min} \cdot \dfrac{1 \text{ hr}}{60 \text{ min}} = 0.4 \text{ hr}$
$F(8.4) = 5\,\text{int}(8.4+1)+1 = 5(9)+1 = 46$
Parking for 8 hours and 24 minutes costs \$46.

47. **a.** Charge for 50 therms:

$C = 9.45 + 0.6338(50) + 0.36375(50)$
$= \$59.33$

b. Charge for 500 therms:

$C = 9.45 + 0.36375(50) + 0.11445(450)$
$+ 0.6338(500)$
$= \$396.04$

c. For $0 \le x \le 50$:

$C = 9.45 + 0.36375x + 0.6338x$
$= 9.45 + 0.99755x$

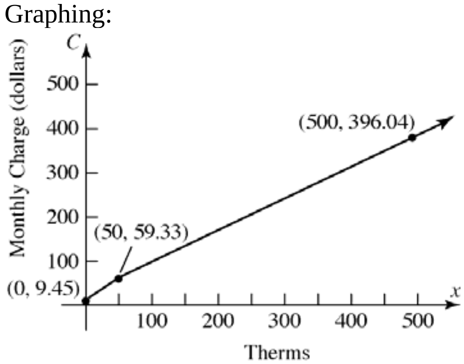
For $x > 50$:

$C = 9.45 + 0.36375(50) + 0.11445(x - 50)$
$+ 0.6338x$
$= 9.45 + 18.1875 + 0.11445x - 5.7225$
$+ 0.6338x$
$= 21.915 + 0.74825x$

The monthly charge function:

$$C = \begin{cases} 9.45 + 0.99755x & \text{for } 0 \le x \le 50 \\ 21.915 + 0.74825x & \text{for } x > 50 \end{cases}$$

d. Graphing:



48. **a.** Charge for 40 therms:

$C = 6.45 + 0.2012(20) + 0.1117(20)$
$+ 0.7268(40)$
$= \$41.78$

b. Charge for 202 therms:

$C = 6.45 + 0.2012(20) + 0.1117(30)$
$+ 0.0374(152) + 0.7268(202)$
$= \$166.32$

c. For $0 \le x \le 20$:

$C = 6.45 + 0.2012x + 0.7268x$
$= 6.45 + 0.928x$

For $20 < x \le 50$:

$C = 6.45 + 0.2012(20) + 0.1117(x - 20)$
$+ 0.7268x$
$= 6.45 + 4.024 + 0.1117x - 2.234$
$+ 0.7268x$
$= 8.24 + 0.8385x$

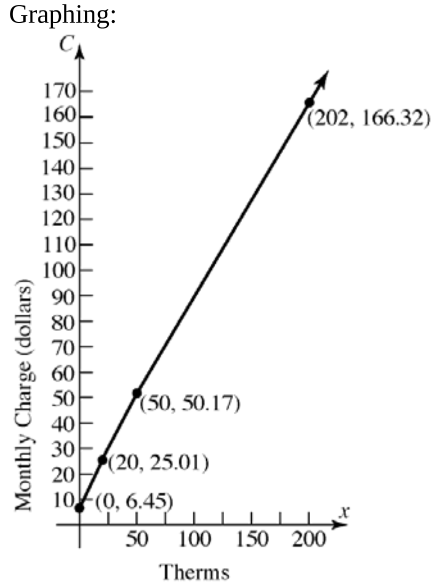
For $x > 50$:

$C = 6.45 + 0.2012(20) + 0.1117(30)$
$+ 0.0374(x - 50) + 0.7268x$
$= 6.45 + 4.024 + 3.351 + 0.0374x - 1.87$
$+ 0.7268x$
$= 11.955 + 0.7642x$

The monthly charge function:

$$C(x) = \begin{cases} 0.928x & \text{if } 0 \le x \le 20 \\ 0.8385x + 8.24 & \text{if } 20 < x \le 50 \\ 0.7642x + 11.955 & \text{if } x > 50 \end{cases}$$

d. Graphing:



49. For schedule X:

$$f(x)=\begin{cases}0.10x & \text{if } 0<x\le 7150\\ 715+0.15(x-7150) & \text{if } 7150<x\le 29,050\\ 4000+0.25(x-29,050) & \text{if } 29,050<x\le 70,350\\ 14,325+0.28(x-70,350) & \text{if } 70,350<x\le 146,750\\ 35,717+0.33(x-146,750) & \text{if } 146,750<x\le 319,100\\ 92,592.50+0.35(x-319,100) & \text{if } x>319,100\end{cases}$$

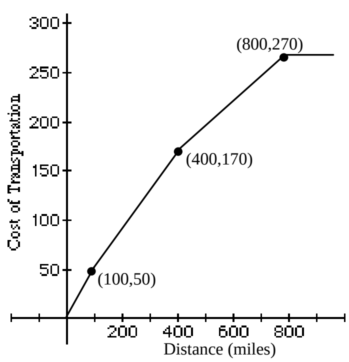
50. For Schedule *Y*-1:

$$f(x)=\begin{cases}0.10x & \text{if } 0<x\le 14,300\\ 1430+0.15(x-14,300) & \text{if } 14,300<x\le 58,100\\ 8000+0.25(x-58,100) & \text{if } 58,100<x\le 117,250\\ 22,787.50+0.28(x-117,250) & \text{if } 117,250<x\le 178,650\\ 39,979.50+0.33(x-178,650) & \text{if } 178,650<x\le 319,100\\ 86,328.00+0.35(x-319,100) & \text{if } x>319,100\end{cases}$$

51. **a.** Let x represent the number of miles and C be the cost of transportation.

$$C(x)=\begin{cases}0.50x & \text{if } 0\le x\le 100\\ 0.50(100)+0.40(x-100) & \text{if } 100<x\le 400\\ 0.50(100)+0.40(300)+0.25(x-400) & \text{if } 400<x\le 800\\ 0.50(100)+0.40(300)+0.25(400)+0(x-800) & \text{if } 800<x\le 960\end{cases}$$

$$C(x)=\begin{cases}0.50x & \text{if } 0\le x\le 100\\ 10+0.40x & \text{if } 100<x\le 400\\ 70+0.25x & \text{if } 400<x\le 800\\ 270 & \text{if } 800<x\le 960\end{cases}$$

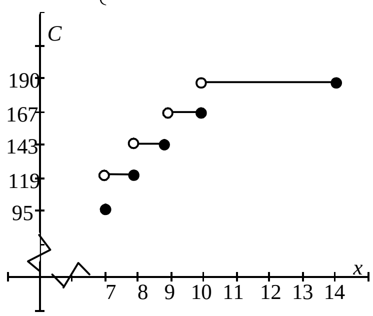


b. For hauls between 100 and 400 miles the cost is: $C(x)=10+0.40x$.

c. For hauls between 400 and 800 miles the cost is: $C(x)=70+0.25x$.

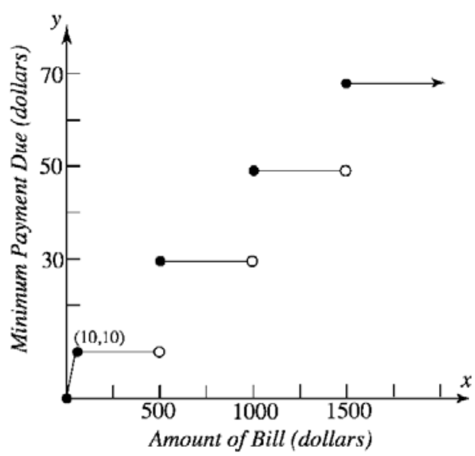
52. Let x = number of days car is used. The cost of renting is given by

$$C(x) = \begin{cases} 95 & \text{if } x = 7 \\ 119 & \text{if } 7 < x \le 8 \\ 143 & \text{if } 8 < x \le 9 \\ 167 & \text{if } 9 < x \le 10 \\ 190 & \text{if } 10 < x \le 14 \end{cases}$$



53. Let x = the amount of the bill in dollars. The minimum payment due is given by

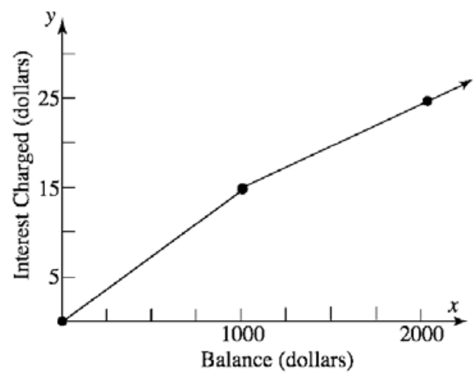
$$f(x) = \begin{cases} x & \text{if } x < 10 \\ 10 & \text{if } 10 \le x < 500 \\ 30 & \text{if } 500 \le x < 1000 \\ 50 & \text{if } 1000 \le x < 1500 \\ 70 & \text{if } 1500 \le x \end{cases}$$



54. Let x = the balance of the bill in dollars. The monthly interest charge is given by

$$g(x) = \begin{cases} 0.015x & \text{if } x \le 1000 \\ 15 + 0.01(x - 1000) & \text{if } 1000 < x \end{cases}$$

$$= \begin{cases} 0.015x & \text{if } x \le 1000 \\ 5 + 0.01x & \text{if } 1000 < x \end{cases}$$



55. **a.** $W = 10°C$

b. $W = 33 - \dfrac{(10.45 + 10\sqrt{5} - 5)(33 - 10)}{22.04}$

$\approx 3.98°C$

c. $W = 33 - \dfrac{(10.45 + 10\sqrt{15} - 15)(33 - 10)}{22.04}$

$\approx -2.67°C$

d. $W = 33 - 1.5958(33 - 10) = -3.7°C$

e. When $0 \le v < 1.79$, the wind speed is so small that there is no effect on the temperature.

f. For each drop of 1˚ in temperature, the wind chill factor drops approximately 1.6˚C. When the wind speed exceeds 20, there is a constant drop in temperature. That is, the windchill depends only on the temperature.

56. **a.** $W = -10°C$

b. $W = 33 - \dfrac{\left(10.45 + 10\sqrt{5} - 5\right)\left(33 - (-10)\right)}{22.04}$

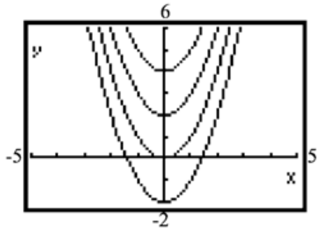
$\approx -21.26°C$

c. $W = 33 - \dfrac{\left(10.45 + 10\sqrt{15} - 15\right)\left(33 - (-10)\right)}{22.04}$

$\approx -33.68°C$

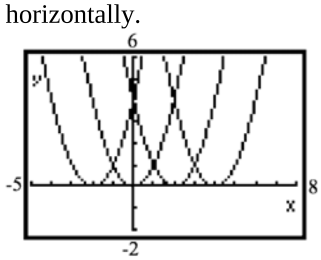
d. $W = 33 - 1.5958\left(33 - (-10)\right) = -35.62°C$

57. Each graph is that of $y = x^2$, but shifted vertically.



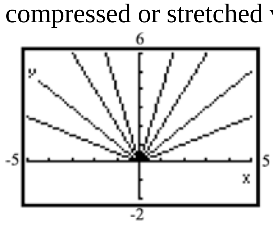
If $y = x^2 + k,\ k > 0$, the shift is up k units; if $y = x^2 + k,\ k < 0$, the shift is down $|k|$ units. The graph of $y = x^2 - 4$ is the same as the graph of $y = x^2$, but shifted down 4 units. The graph of $y = x^2 + 5$ is the graph of $y = x^2$, but shifted up 5 units.

58. Each graph is that of $y = x^2$, but shifted horizontally.



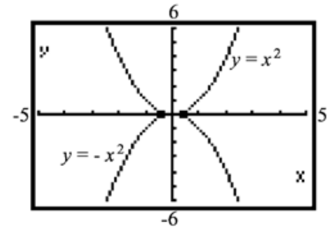
If $y = (x - k)^2,\ k > 0$, the shift is to the right k units; if $y = (x - k)^2,\ k < 0$, the shift is to the left $|k|$ units. The graph of $y = (x + 4)^2$ is the same as the graph of $y = x^2$, but shifted to the left 4 units. The graph of $y = (x - 5)^2$ is the graph of $y = x^2$, but shifted to the right 5 units.

59. Each graph is that of $y = |x|$, but either compressed or stretched vertically.

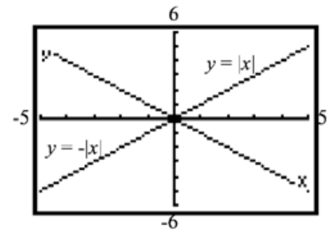


If $y = k|x|$ and $k > 1$, the graph is stretched; if $y = k|x|$ and $0 < k < 1$, the graph is compressed. The graph of $y = \frac{1}{4}|x|$ is the same as the graph of $y = |x|$, but compressed. The graph of $y = 5|x|$ is the same as the graph of $y = |x|$, but stretched.

60. The graph of $y = -x^2$ is the reflection of the graph of $y = x^2$ about the *x*-axis.

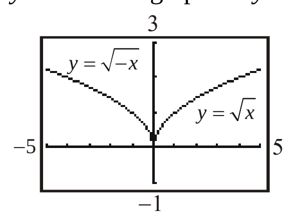


The graph of $y = -|x|$ is the reflection of the graph of $y = |x|$ about the *x*-axis.

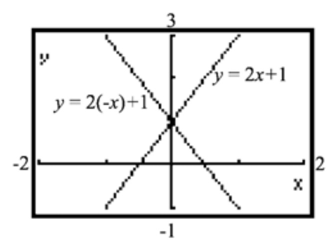


Multiplying a function by –1 causes the graph to be a reflection about the *x*-axis of the original function's graph.

61. The graph of $y = \sqrt{-x}$ is the reflection about the *y*-axis of the graph of $y = \sqrt{x}$.

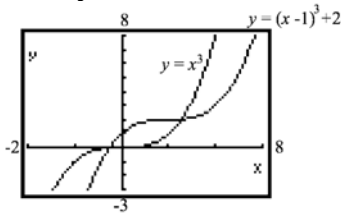


The same type of reflection occurs when graphing $y = 2x + 1$ and $y = 2(-x) + 1$.

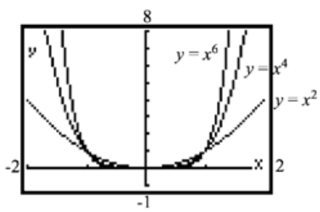


The graph of $y = f(-x)$ is the reflection about the *y*-axis of the graph of $y = f(x)$.

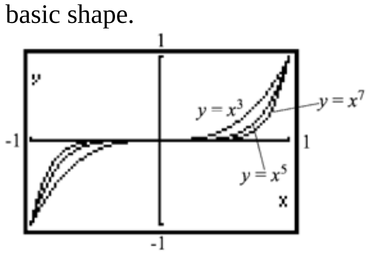
62. The graph of $y=(x-1)^3+2$ is a shifting of the graph of $y=x^3$ one unit to the right and two units up.



63. For the graph of $y=x^n$, n a positive even integer, as n increases, the graph of the function is narrower for $|x|>1$ and flatter for $|x|<1$.



64. For the graph of $y=x^n$, n a positive odd integer, as n increases, the graph of the function increases at a greater rate for $|x|>1$ and is flatter around 0 for $|x|<1$. They have the same basic shape.



65. Yes, it is a function.

$$f(x)=\begin{cases}1 & \text{if } x \text{ is rational}\\ 0 & \text{if } x \text{ is irrational}\end{cases}$$

$\{x \mid x \text{ is any real number}\}$ Range = {0,1}

y-intercept: $x=0 \Rightarrow x$ is rational $\Rightarrow y=1$ So the *y*-intercept is (0, 1).

x-intercept: $y=0 \Rightarrow x$ is irrational So the graph has infinitely many *x*-intercepts, namely, there is an *x*-intercept at each irrational value of *x*.

$f(-x)=1=f(x)$ when *x* is rational;

$f(-x)=0=f(x)$ when *x* is irrational, So f is even.

The graph of f consists of 2 infinite clusters of distinct points, extending horizontally in both directions. One cluster is located 1 unit above the *x*-axis, and the other is located along the *x*-axis.

66. For 0 < *x* < 1, the graph of $y=x^r$, *r* rational and $r>0$, flattens down toward the *x*-axis as *r* gets bigger.

For 1 < *x*, the graph of $y=x^r$ increases at a greater rate as *r* gets bigger.

Section 2.6

1. horizontal; right
2. y-axis
3. -5, -2, and 2
4. True; the graph of $y=-f(x)$ is the reflection about the x-axis of the graph of $y=f(x)$.
5. False; to obtain the graph of $y=f(x+2)-3$ you shift the graph of $y=f(x)$ to the left 2 units and down 3 units.
6. True; to obtain the graph of $y=2f(x)$ we multiply the y-coordinates of the graph of $y=f(x)$ by 2. Since the y-coordinate of x-intercepts is 0 and $2\cdot 0=0$, multiplying by a constant does not change the x-intercepts.
7. B
8. E
9. H
10. D
11. I
12. A
13. L
14. C
15. F

16. J

17. G

18. K

19. $y=(x-4)^3$

20. $y=(x+4)^3$

21. $y=x^3+4$

22. $y=x^3-4$

23. $y=(-x)^3=-x^3$

24. $y=-x^3$

25. $y=4x^3$

26. $y=\left(\frac{1}{4}x\right)^3=\frac{x^3}{64}$

27. (1) $y=\sqrt{x}+2$
(2) $y=-\left(\sqrt{x}+2\right)$
(3) $y=-\left(\sqrt{-x}+2\right)=-\sqrt{-x}-2$

28. (1) $y=-\sqrt{x}$
(2) $y=-\sqrt{x-3}$
(3) $y=-\sqrt{x-3}-2$

29. (1) $y=-\sqrt{x}$
(2) $y=-\sqrt{x}+2$
(3) $y=-\sqrt{x+3}+2$

30. (1) $y=\sqrt{x}+2$
(2) $y=\sqrt{-x}+2$
(3) $y=\sqrt{-(x+3)}+2=\sqrt{-x-3}+2$

31. (c); To go from $y=f(x)$ to $y=-f(x)$ we reflect about the x-axis. This means we change the sign of the y-coordinate for each point on the graph of $y=f(x)$. Thus, the point $(3,0)$ would remain the same.

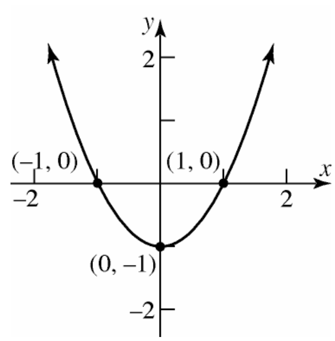
32. (d); To go from $y=f(x)$ to $y=f(-x)$, we reflect each point on the graph of $y=f(x)$ about the y-axis. This means we change the sign of the x-coordinate for each point on the graph of $y=f(x)$. Thus, the point $(3,0)$ would become $(-3,0)$.

33. (c); To go from $y=f(x)$ to $y=2f(x)$, we multiply the y-coordinate of each point on the graph of $y=f(x)$ by 2. Thus, the point $(0,3)$ would become $(0,6)$.

34. (a); To go from $y=f(x)$ to $y=\frac{1}{2}f(x)$, we multiply the y-coordinate of each point on the graph of $y=f(x)$ by $\frac{1}{2}$. Thus, the point $(3,0)$ would remain $(3,0)$.

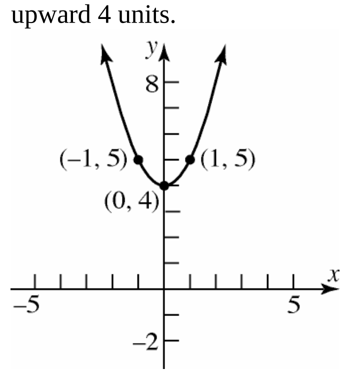
35. $f(x)=x^2-1$

Using the graph of $y=x^2$, vertically shift downward 1 unit.



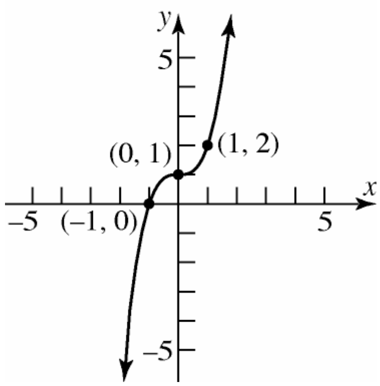
36. $f(x)=x^2+4$

Using the graph of $y=x^2$, vertically shift upward 4 units.



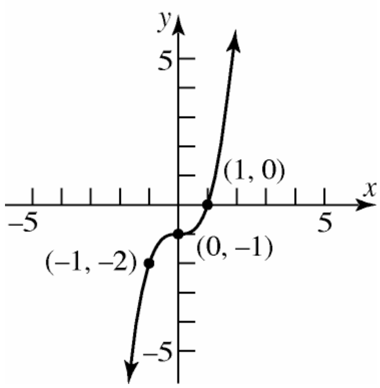
37. $g(x) = x^3 + 1$

Using the graph of $y = x^3$, vertically shift upward 1 unit.



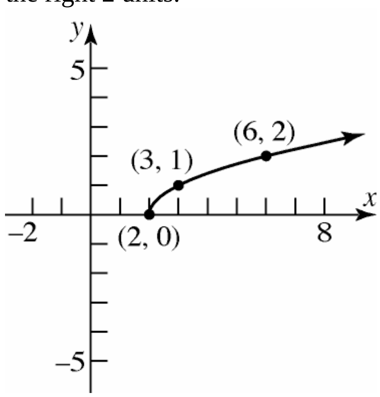
38. $g(x) = x^3 - 1$

Using the graph of $y = x^3$, vertically shift downward 1 unit.



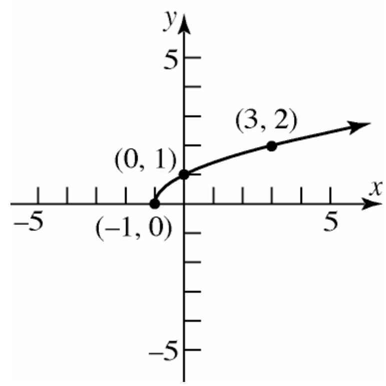
39. $h(x) = \sqrt{x-2}$

Using the graph of $y = \sqrt{x}$, horizontally shift to the right 2 units.



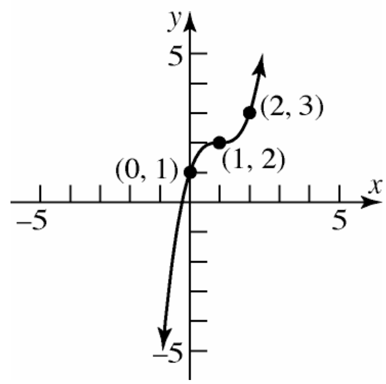
40. $h(x) = \sqrt{x+1}$

Using the graph of $y = \sqrt{x}$, horizontally shift to the left 1 unit.



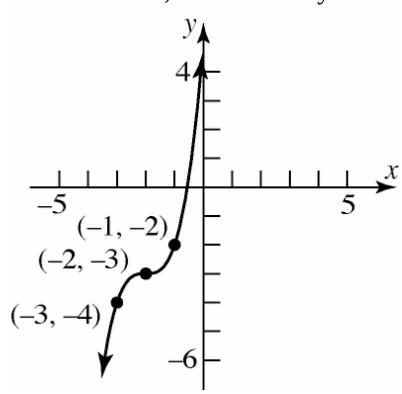
41. $f(x) = (x-1)^3 + 2$

Using the graph of $y = x^3$, horizontally shift to the right 1 unit, then vertically shift up 2 units.



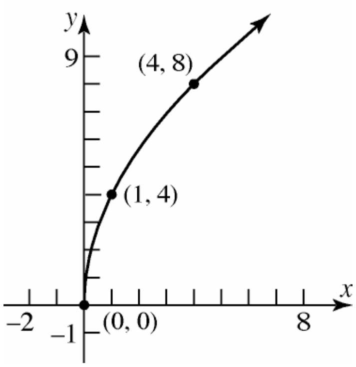
42. $f(x) = (x+2)^3 - 3$

Using the graph of $y = x^3$, horizontally shift to the left 2 units, then vertically shift down 3 units.



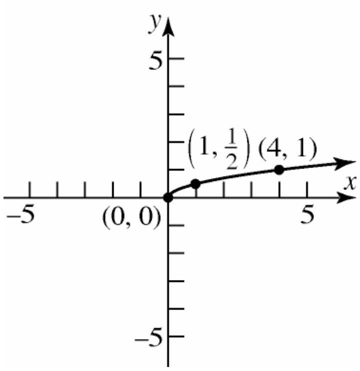
43. $g(x)=4\sqrt{x}$

Using the graph of $y=\sqrt{x}$, vertically stretch by a factor of 4.



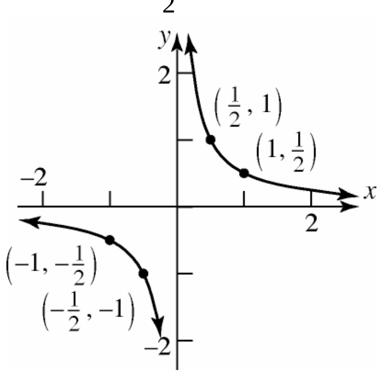
44. $g(x)=\frac{1}{2}\sqrt{x}$

Using the graph of $y=\sqrt{x}$, vertically compress by a factor of $\frac{1}{2}$.



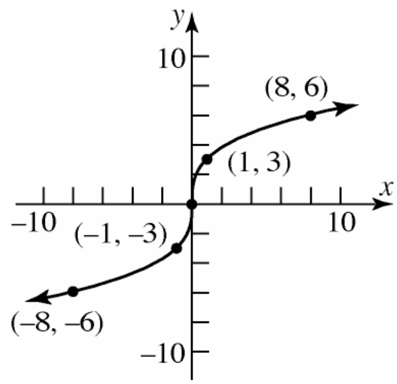
45. $h(x)=\frac{1}{2x}=\left(\frac{1}{2}\right)\left(\frac{1}{x}\right)$

Using the graph of $y=\frac{1}{x}$, vertically compress by a factor of $\frac{1}{2}$.



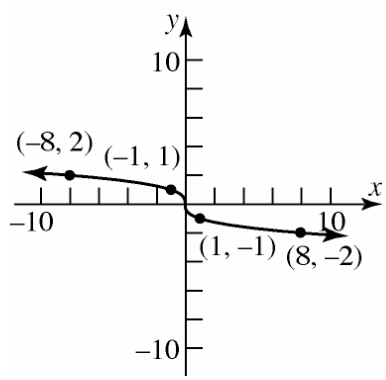
46. $h(x)=3\sqrt[3]{x}$

Using the graph of $y=\sqrt[3]{x}$, vertically stretch by a factor of 3.



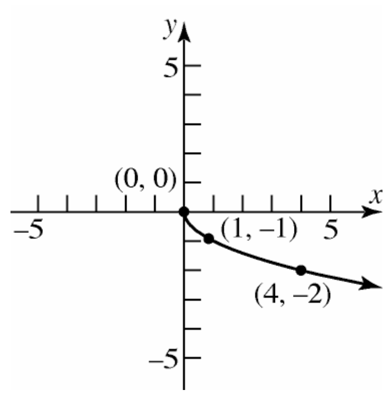
47. $f(x)=-\sqrt[3]{x}$

Reflect the graph of $y=\sqrt[3]{x}$, about the *x*-axis.



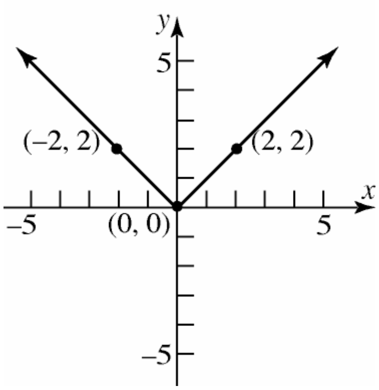
48. $f(x)=-\sqrt{x}$

Reflect the graph of $y=\sqrt{x}$, about the *x*-axis.



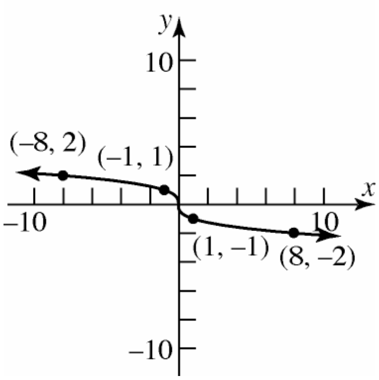
49. $g(x) = |-x|$

Reflect the graph of $y = |x|$ about the y-axis.



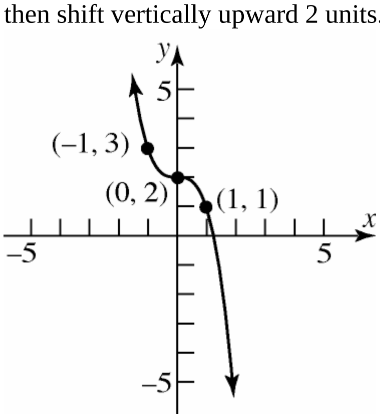
50. $g(x) = \sqrt[3]{-x}$

Reflect the graph of $y = \sqrt[3]{x}$, about the y-axis.



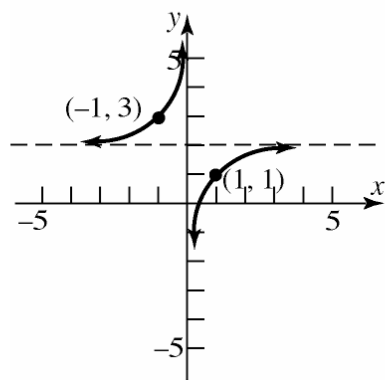
51. $h(x) = -x^3 + 2$

Reflect the graph of $y = x^3$ about the x-axis, then shift vertically upward 2 units.



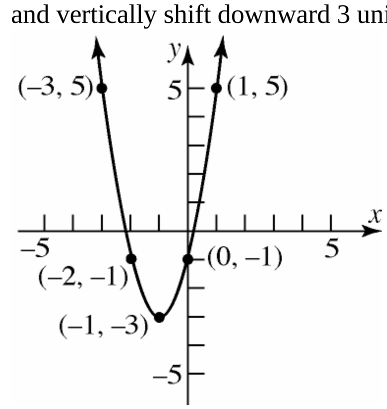
52. $h(x) = \dfrac{1}{-x} + 2$

Reflect the graph of $y = \dfrac{1}{x}$ about the y-axis, then shift vertically upward 2 units.



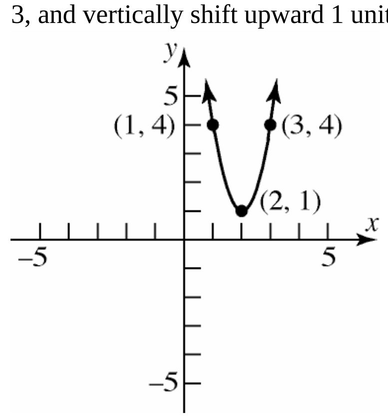
53. $f(x) = 2(x+1)^2 - 3$

Using the graph of $y = x^2$, horizontally shift to the left 1 unit, vertically stretch by a factor of 2, and vertically shift downward 3 units.



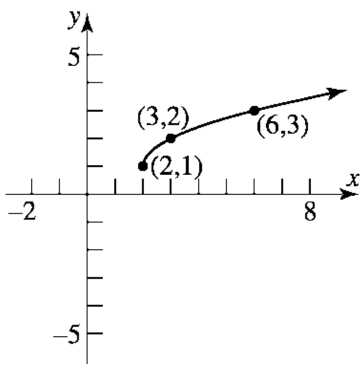
54. $f(x) = 3(x-2)^2 + 1$

Using the graph of $y = x^2$, horizontally shift to the right 2 units, vertically stretch by a factor of 3, and vertically shift upward 1 unit.



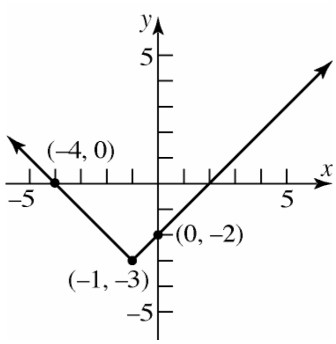
55. $g(x) = \sqrt{x-2} + 1$

Using the graph of $y = \sqrt{x}$, horizontally shift to the right 2 units and vertically shift upward 1 unit.



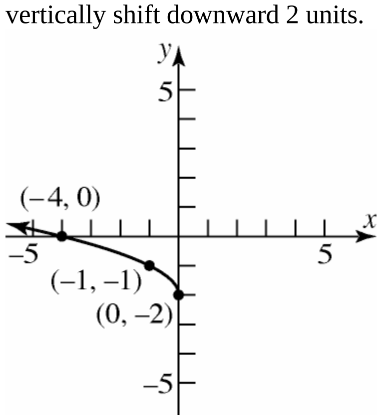
56. $g(x) = |x+1| - 3$

Using the graph of $y = |x|$, horizontally shift to the left 1 unit and vertically shift downward 3 units.



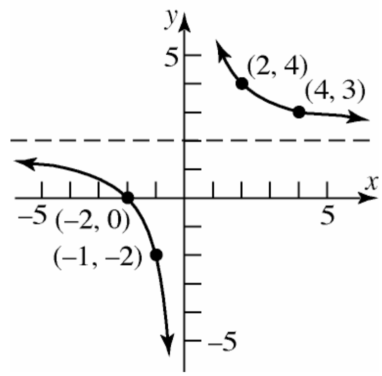
57. $h(x) = \sqrt{-x} - 2$

Reflect the graph of $y = \sqrt{x}$ about the *y*-axis and vertically shift downward 2 units.



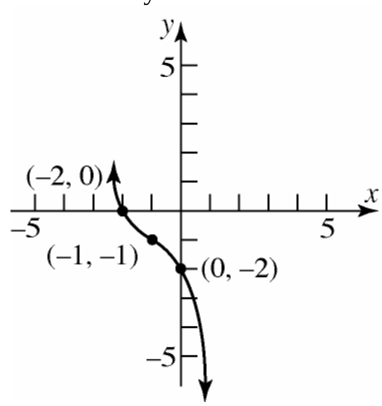
58. $h(x) = \frac{4}{x} + 2 = 4\left(\frac{1}{x}\right) + 2$

Stretch the graph of $y = \frac{1}{x}$ vertically by a factor of 4 and vertically shift upward 2 units.



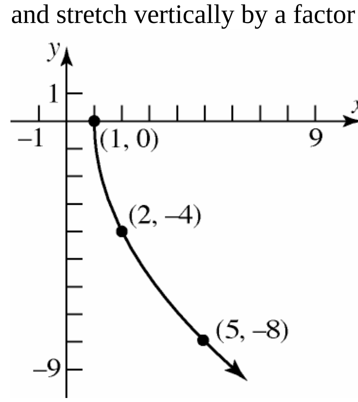
59. $f(x) = -(x+1)^3 - 1$

Using the graph of $y = x^3$, horizontally shift to the left 1 unit, reflect the graph about the *x*-axis, and vertically shift downward 1 unit.



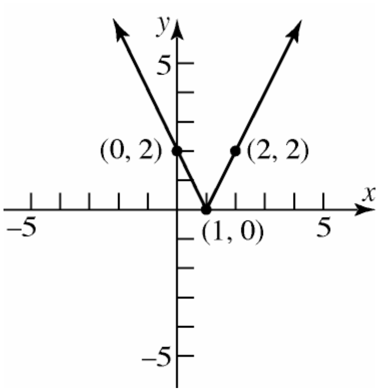
60. $f(x) = -4\sqrt{x-1}$

Using the graph of $y = \sqrt{x}$, horizontally shift to the right 1 unit, reflect the graph about the *x*-axis, and stretch vertically by a factor of 4.



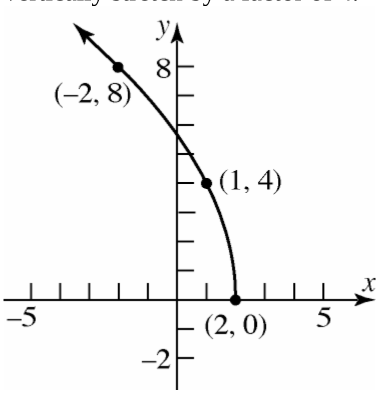
61. $g(x) = 2|1-x| = 2|-(-1+x)| = 2|x-1|$

Using the graph of $y = |x|$, horizontally shift to the right 1 unit, and vertically stretch by a factor or 2.



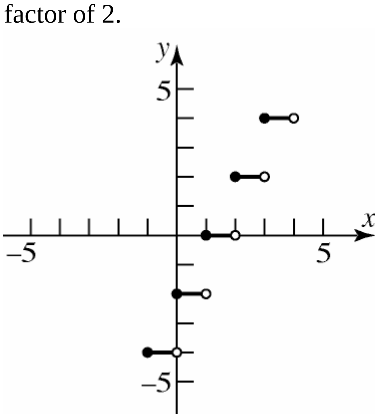
62. $g(x) = 4\sqrt{2-x} = 4\sqrt{-(x-2)}$

Reflect the graph of $y = \sqrt{x}$ about the y-axis, horizontally shift to the right 2 units, and vertically stretch by a factor of 4.



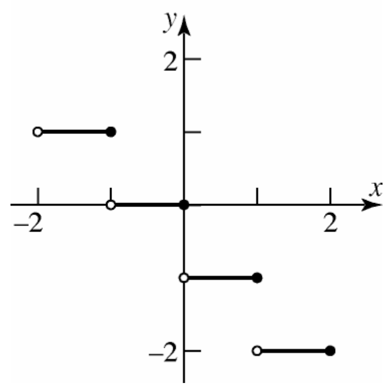
63. $h(x) = 2\,\text{int}(x-1)$

Using the graph of $y = \text{int}(x)$, horizontally shift to the right 1 unit, and vertically stretch by a factor of 2.



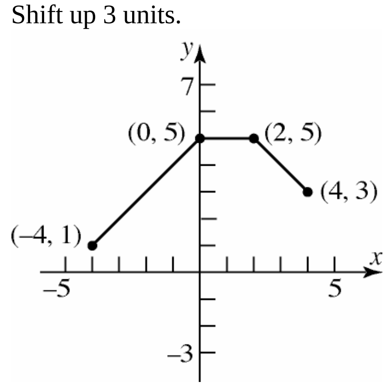
64. $h(x) = \text{int}(-x)$

Reflect the graph of $y = \text{int}(x)$ about the y-axis.



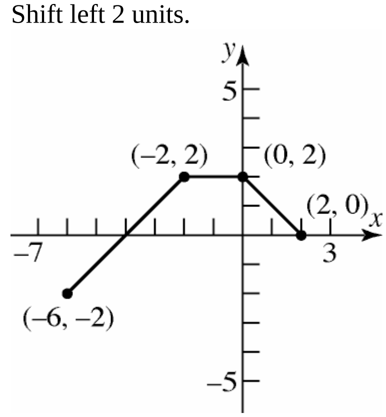
65. a. $F(x) = f(x)+3$

Shift up 3 units.

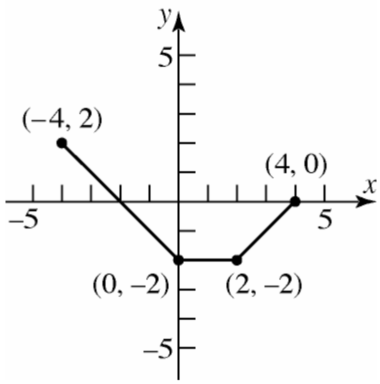


b. $G(x) = f(x+2)$

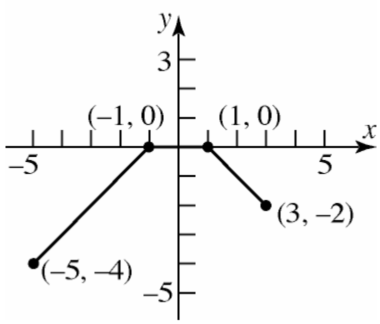
Shift left 2 units.



c. $P(x) = -f(x)$
Reflect about the *x*-axis.

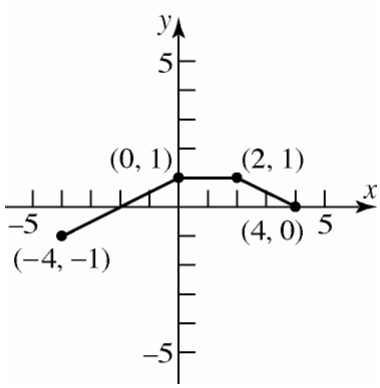


d. $H(x) = f(x+1) - 2$
Shift left 1 unit and shift down 2 units.

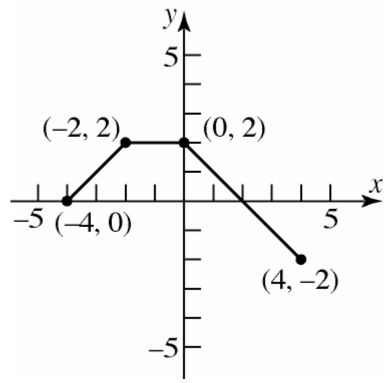


e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.

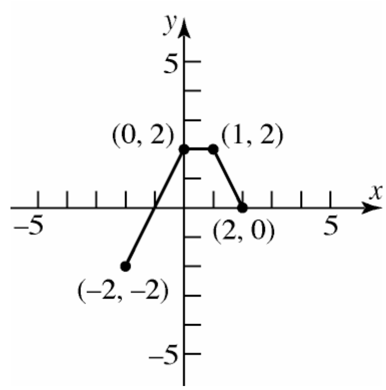


f. $g(x) = f(-x)$
Reflect about the *y*-axis.

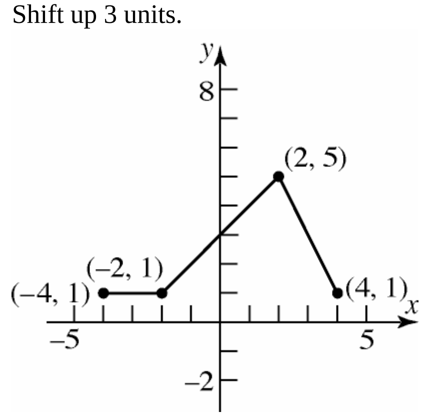


g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.

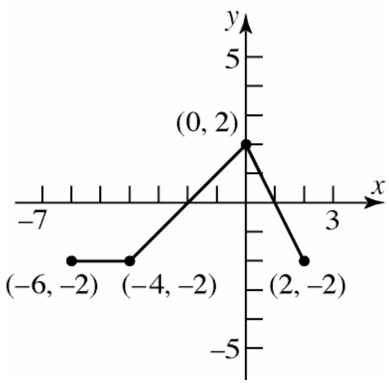


66. a. $F(x) = f(x) + 3$
Shift up 3 units.



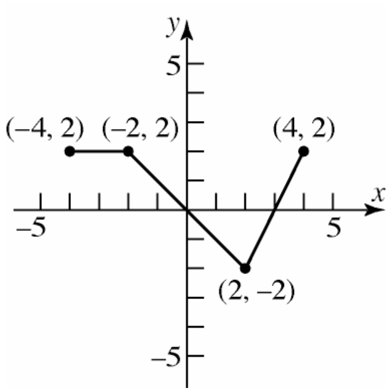
b. $G(x) = f(x+2)$

Shift left 2 units.



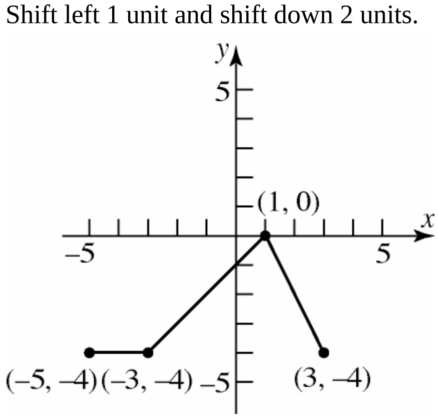
c. $P(x) = -f(x)$

Reflect about the *x*-axis.



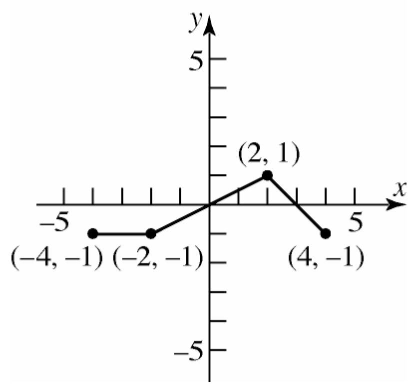
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



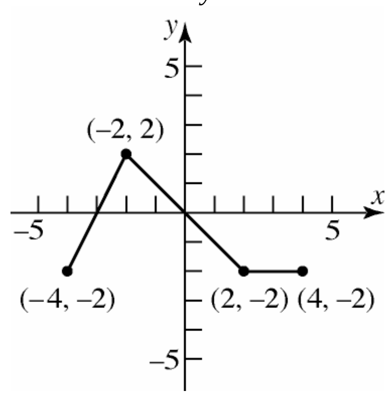
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



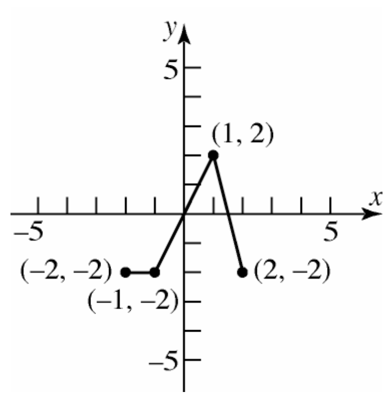
f. $g(x) = f(-x)$

Reflect about the *y*-axis.



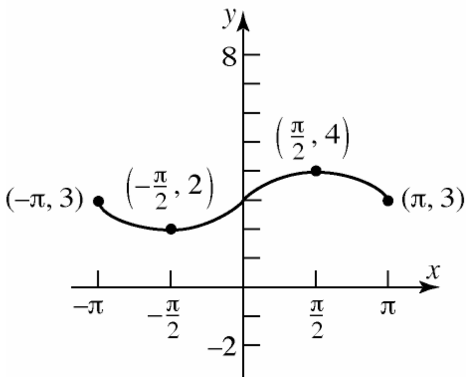
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



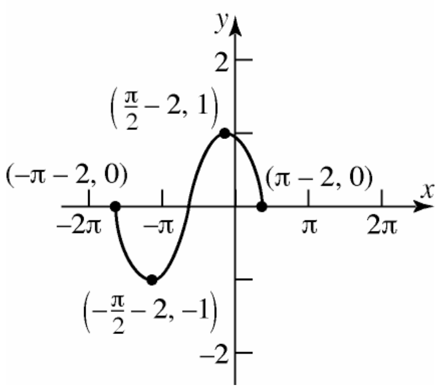
67. a. $F(x) = f(x) + 3$

Shift up 3 units.



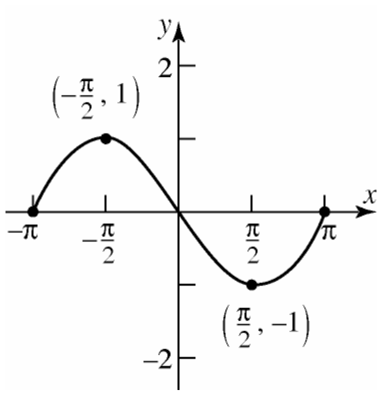
b. $G(x) = f(x+2)$

Shift left 2 units.



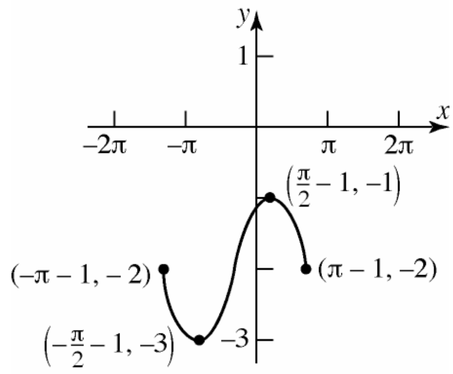
c. $P(x) = -f(x)$

Reflect about the *x*-axis.



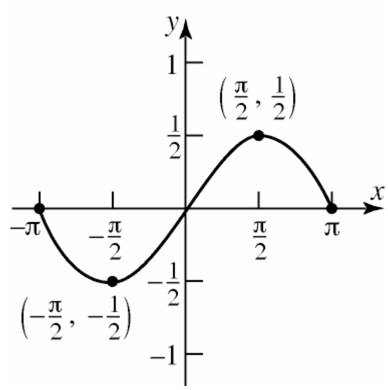
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



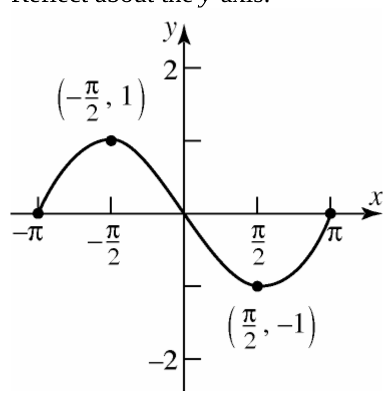
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



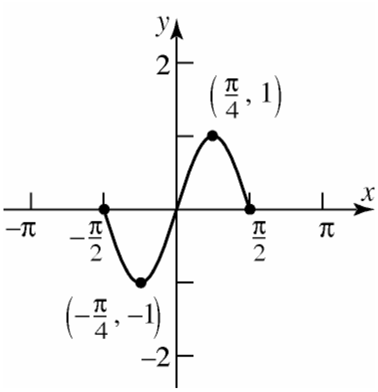
f. $g(x) = f(-x)$

Reflect about the *y*-axis.



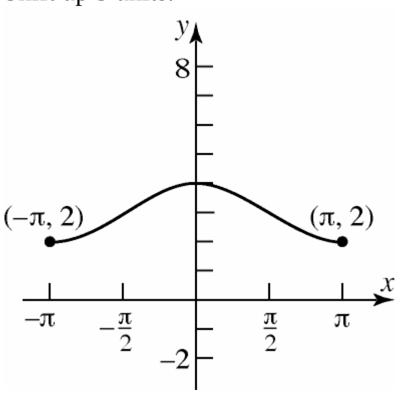
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



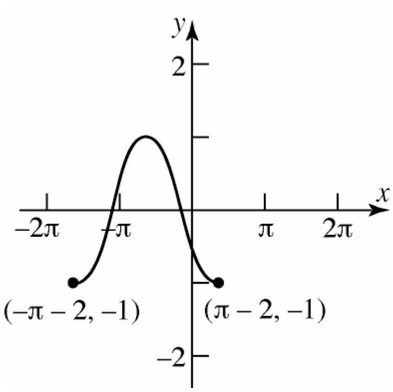
68. a. $F(x) = f(x) + 3$

Shift up 3 units.



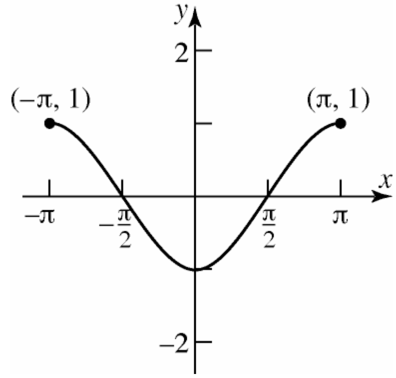
b. $G(x) = f(x+2)$

Shift left 2 units.



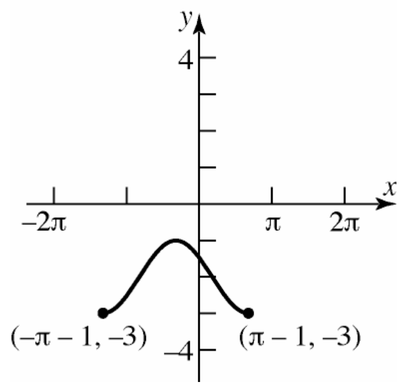
c. $P(x) = -f(x)$

Reflect about the x-axis.



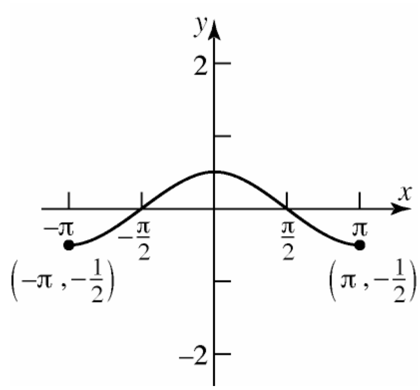
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.

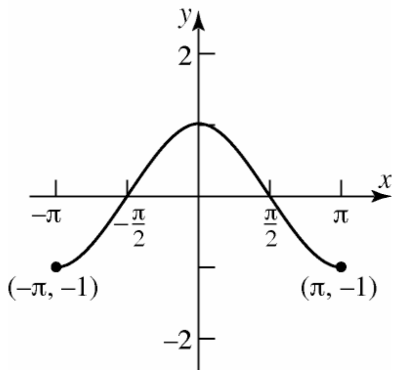


e. $Q(x) = \frac{1}{2} f(x)$

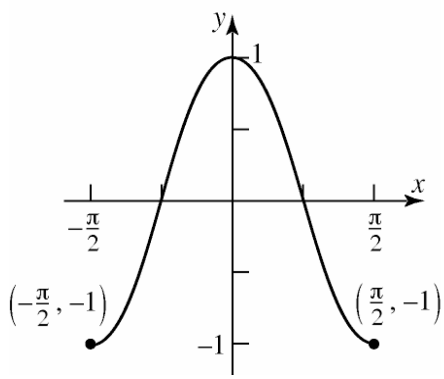
Compress vertically by a factor of $\frac{1}{2}$.



f. $g(x) = f(-x)$
Reflect about the *y*-axis.



g. $h(x) = f(2x)$
Compress horizontally by a factor of $\frac{1}{2}$.



69. a. The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the x-intercepts are -7 and 1.

b. The graph of $y = f(x-2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the right. Therefore, the x-intercepts are -3 and 5.

c. The graph of $y = 4f(x)$ is the same as the graph of $y = f(x)$, but stretched vertically by a factor of 4. Therefore, the x-intercepts are still -5 and 3 since the y-coordinate of each is 0.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, the x-intercepts are 5 and -3.

70. a. The graph of $y = f(x+4)$ is the same as the graph of $y = f(x)$, but shifted 4 units to the left. Therefore, the x-intercepts are -12 and -3.

b. The graph of $y = f(x-3)$ is the same as the graph of $y = f(x)$, but shifted 3 units to the right. Therefore, the x-intercepts are -5 and 4.

c. The graph of $y = 2f(x)$ is the same as the graph of $y = f(x)$, but stretched vertically by a factor of 2. Therefore, the x-intercepts are still -8 and 1 since the y-coordinate of each is 0.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, the x-intercepts are 8 and -1.

71. a. The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is increasing on the interval $(-3,3)$.

b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is increasing on the interval $(4,10)$.

c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y = -f(x)$ must be *decreasing* on the interval $(-1,5)$.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y = f(-x)$ must be *decreasing* on the interval $(-5,1)$.

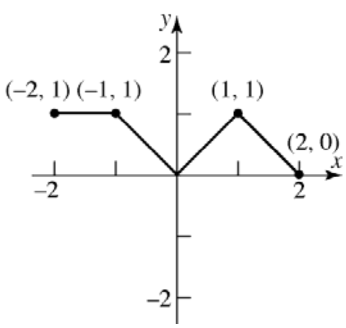
72. a. The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is decreasing on the interval $(-4, 5)$.

b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is decreasing on the interval $(3, 12)$.

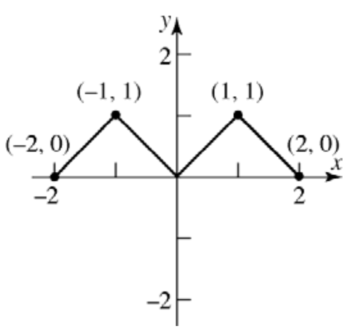
c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y = -f(x)$ must be *increasing* on the interval $(-2, 7)$.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y = f(-x)$ must be *increasing* on the interval $(-7, 2)$.

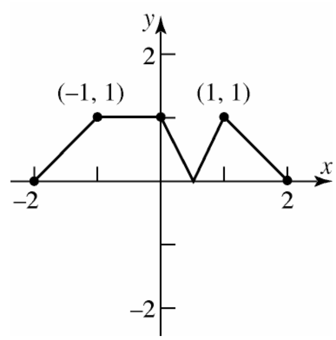
73. a. $y = |f(x)|$



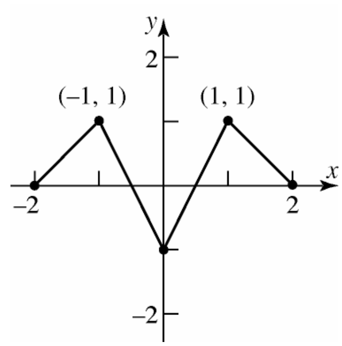
b. $y = f(|x|)$



74. a. $y = |f(x)|$



b. $y = f(|x|)$

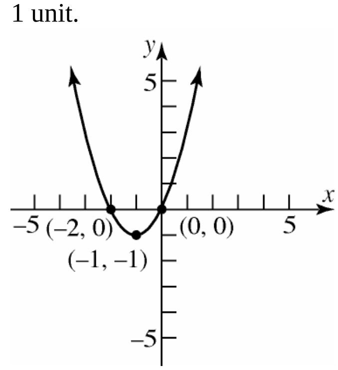


75. $f(x) = x^2 + 2x$

$f(x) = (x^2 + 2x + 1) - 1$

$f(x) = (x+1)^2 - 1$

Using $f(x) = x^2$, shift left 1 unit and shift down 1 unit.

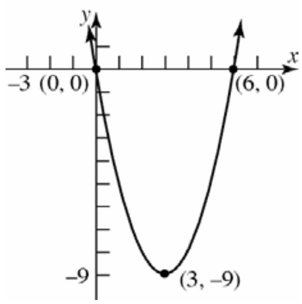


76. $f(x) = x^2 - 6x$

$f(x) = (x^2 - 6x + 9) - 9$

$f(x) = (x-3)^2 - 9$

Using $f(x) = x^2$, shift right 3 units and shift down 9 units.

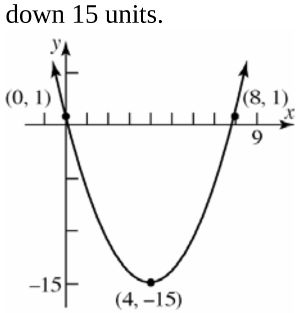


77. $f(x) = x^2 - 8x + 1$

$f(x) = \left(x^2 - 8x + 16\right) + 1 - 16$

$f(x) = \left(x-4\right)^2 - 15$

Using $f(x) = x^2$, shift right 4 units and shift down 15 units.

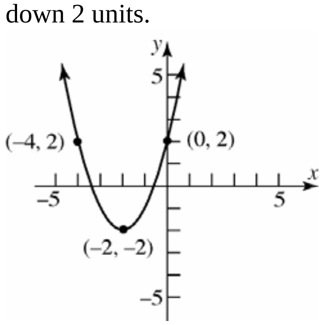


78. $f(x) = x^2 + 4x + 2$

$f(x) = \left(x^2 + 4x + 4\right) + 2 - 4$

$f(x) = \left(x+2\right)^2 - 2$

Using $f(x) = x^2$, shift left 2 units and shift down 2 units.

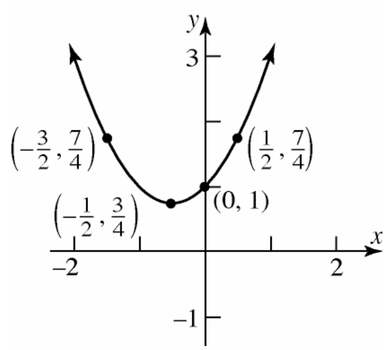


79. $f(x) = x^2 + x + 1$

$f(x) = \left(x^2 + x + \frac{1}{4}\right) + 1 - \frac{1}{4}$

$f(x) = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

Using $f(x) = x^2$, shift left $\frac{1}{2}$ unit and shift up $\frac{3}{4}$ unit.

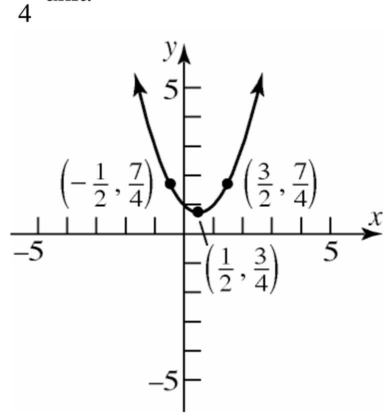


80. $f(x) = x^2 - x + 1$

$f(x) = \left(x^2 - x + \frac{1}{4}\right) + 1 - \frac{1}{4}$

$f(x) = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

Using $f(x) = x^2$, shift right $\frac{1}{2}$ unit and shift up $\frac{3}{4}$ unit.



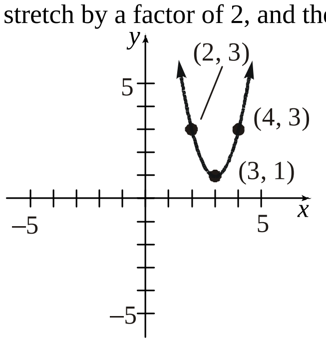
81. $f(x) = 2x^2 - 12x + 19$

$= 2(x^2 - 6x) + 19$

$= 2(x^2 - 6x + 9) + 19 - 18$

$= 2(x-3)^2 + 1$

Using $f(x) = x^2$, shift right 3 units, vertically stretch by a factor of 2, and then shift up 1 unit.



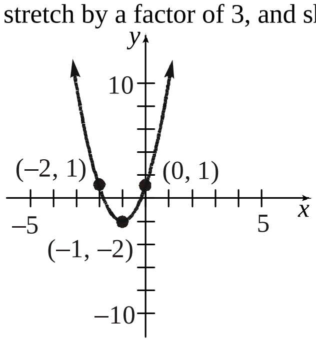
82. $f(x) = 3x^2 + 6x + 1$

$= 3(x^2 + 2x) + 1$

$= 3(x^2 + 2x + 1) + 1 - 3$

$= 3(x+1)^2 - 2$

Using $f(x) = x^2$, shift left 1 unit, vertically stretch by a factor of 3, and shift down 2 units.



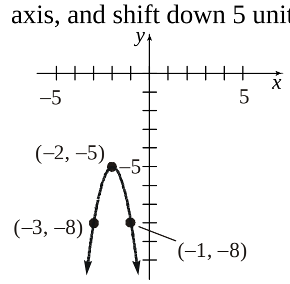
83. $f(x) = -3x^2 - 12x - 17$

$= -3(x^2 + 4x) - 17$

$= -3(x^2 + 4x + 4) - 17 + 12$

$= -3(x+2)^2 - 5$

Using $f(x) = x^2$, shift left 2 units, stretch vertically by a factor of 3, reflect about the x-axis, and shift down 5 units.



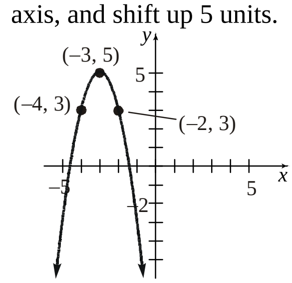
84. $f(x) = -2x^2 - 12x - 13$

$= -2(x^2 + 6x) - 13$

$= -2(x^2 + 6x + 9) - 13 + 18$

$= -2(x+3)^2 + 5$

Using $f(x) = x^2$, shift left 3 units, stretch vertically by a factor of 2, reflect about the x-axis, and shift up 5 units.

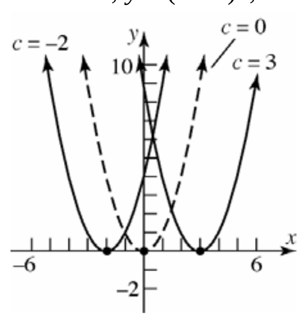


85. $y = (x - c)^2$

If $c = 0,\ y = x^2$.

If $c = 3,\ y = (x-3)^2$; shift right 3 units.

If $c = -2,\ y = (x+2)^2$; shift left 2 units.

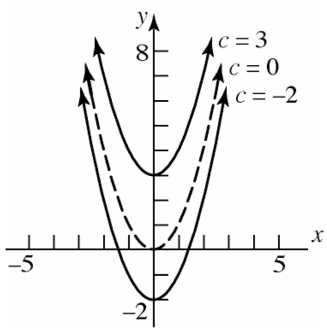


86. $y = x^2 + c$

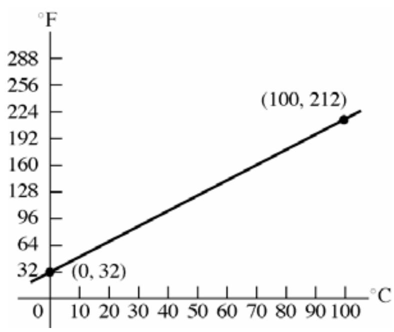
If $c = 0$, $y = x^2$.

If $c = 3$, $y = x^2 + 3$; shift up 3 units.

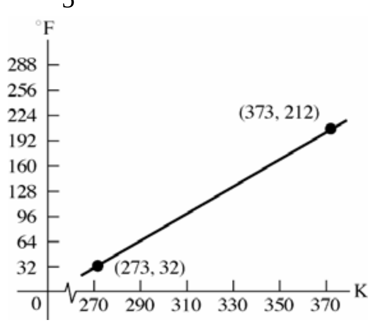
If $c = -2$, $y = x^2 - 2$; shift down 2 units.



87. $F = \frac{9}{5}C + 32$

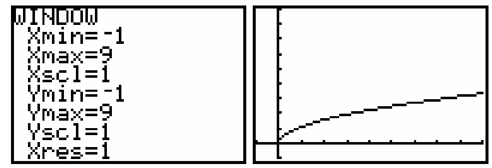


$F = \frac{9}{5}(K - 273) + 32$



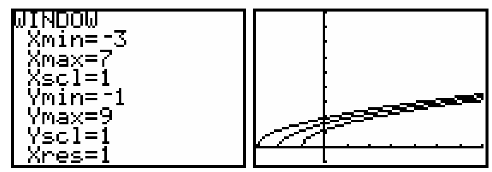
Shift the graph 273 units to the right.

88. a. $T = 2\pi\sqrt{\frac{l}{g}}$



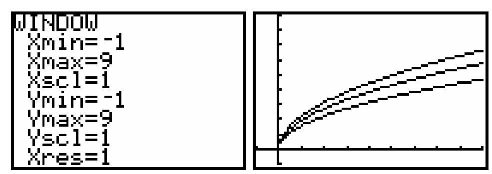
b. $T_1 = 2\pi\sqrt{\frac{l+1}{g}}$; $T_2 = 2\pi\sqrt{\frac{l+2}{g}}$;

$T_3 = 2\pi\sqrt{\frac{l+3}{g}}$



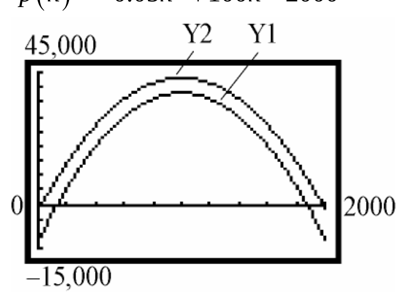
c. As the length of the pendulum increases, the period increases.

d. $T_1 = 2\pi\sqrt{\frac{2l}{g}}$; $T_2 = 2\pi\sqrt{\frac{3l}{g}}$; $T_3 = 2\pi\sqrt{\frac{4l}{g}}$



e. If the length of the pendulum is multiplied by k, the period is multiplied by $\sqrt{k}$.

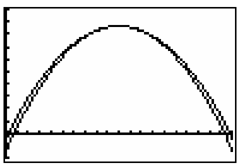
89. a. $p(x) = -0.05x^2 + 100x - 2000$



b. Select the 10% tax since the profits are higher.

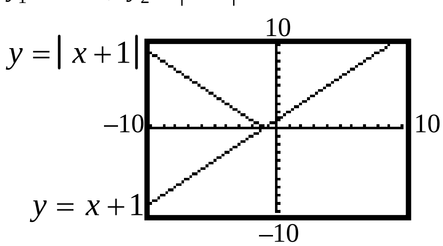
c. The graph of Y1 is obtained by shifting the graph of $p(x)$ vertically down 10,000 units. The graph of Y2 is obtained by multiplying the *y*-coordinate of the graph of $p(x)$ by 0.9. Thus, Y2 is the graph of $p(x)$ vertically compressed by a factor of 0.9.

d. Select the 10% tax since the graph of $Y1 = 0.9p(x) \geq Y2 = -0.05x^2 + 100x - 6800$ for all x in the domain.

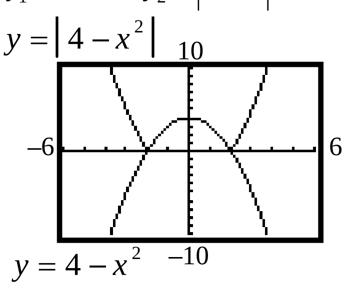


90. The graph of $y = 4f(x)$ is a vertical stretch the graph of f by a factor of 4, while the graph of $y = f(4x)$ is a horizontal compression of the graph of f by a factor of $\frac{1}{4}$.

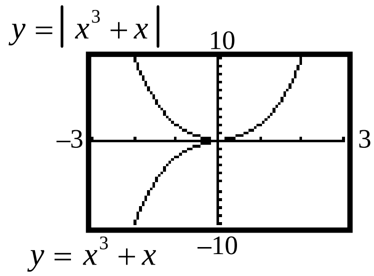
91. a. $y_1 = x+1$; $y_2 = |x+1|$



b. $y_1 = 4 - x^2$; $y_2 = |4 - x^2|$

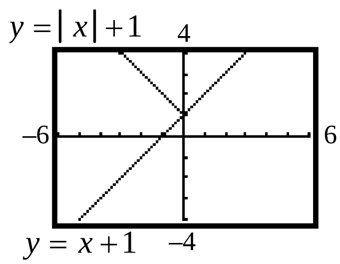


c. $y_1 = x^3 + x$; $y_2 = |x^3 + x|$

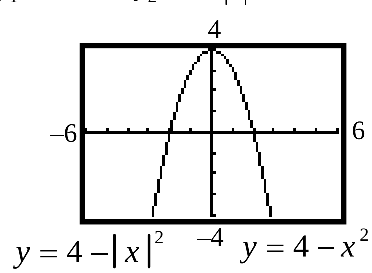


d. Any part of the graph of $y = f(x)$ that lies below the *x*-axis is reflected about the *x*-axis to obtain the graph of $y = |f(x)|$.

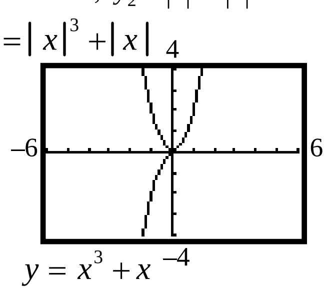
92. a. $y_1 = x+1$; $y_2 = |x|+1$



b. $y_1 = 4 - x^2$; $y_2 = 4 - |x|^2$



c. $y_1 = x^3 + x$; $y_2 = |x|^3 + |x|$



d. The graph of $y = f(|x|)$ is the same as the graph of $y = f(x)$ for $x \ge 0$.

For $x < 0$, the graph of $y = f(|x|)$ is the reflection about the *y*-axis of the graph of $y = f(x)$ when $x > 0$.

Section 2.7

1. $V = \pi r^2 h,\ \ h = 2r \Rightarrow V(r) = \pi r^2 \cdot (2r) = 2\pi r^3$

2. $V = \frac{1}{3}\pi r^2 h,\ \ h = 2r$

$V(r) = \frac{1}{3}\pi \cdot r^2 \cdot (2r) = \frac{2}{3}\pi r^3$

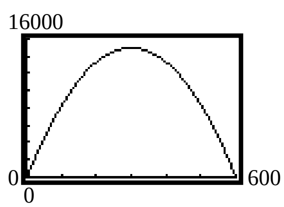
3. a. $R(x) = x\left(-\frac{1}{6}x + 100\right) = -\frac{1}{6}x^2 + 100x$

b. $R(200) = -\frac{1}{6}(200)^2 + 100(200)$

$= \frac{-20{,}000}{3} + 20{,}000$

$= \frac{40{,}000}{3} \approx \$13{,}333.33$

c.



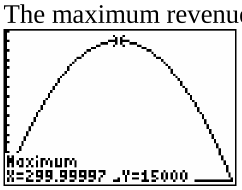
d. $x = 300$ maximizes revenue

$R(300) = -\frac{1}{6}(300)^2 + 100(300)$

$= -15{,}000 + 30{,}000$

$= \$15{,}000$

The maximum revenue is $15,000.



e. $p = -\frac{1}{6}(300) + 100 = -50 + 100 = \$50$ maximizes revenue

4. a. $R(x) = x\left(-\frac{1}{3}x + 100\right)$

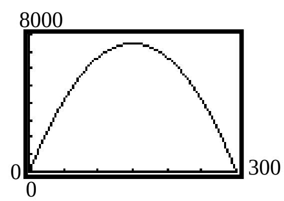
$= -\frac{1}{3}x^2 + 100x$

b. $R(100) = -\frac{1}{3}(100)^2 + 100(100)$

$= \frac{-10{,}000}{3} + 10{,}000$

$= \frac{20{,}000}{3} \approx \$6666.67$

c.



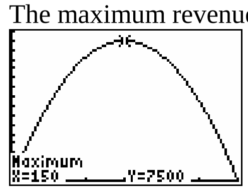
d. $x = 150$ maximizes revenue

$R(150) = -\frac{1}{3}(150)^2 + 100(150)$

$= -7500 + 15{,}000$

$= \$7500$

The maximum revenue is $7500.



e. $p = -\frac{1}{3}(150) + 100 = -50 + 100 = \$50$ maximizes revenue

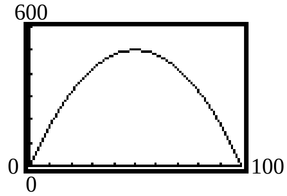
5. a. If $x = -5p + 100$, then $p = \frac{100 - x}{5}$.

$R(x) = x\left(\frac{100 - x}{5}\right) = -\frac{1}{5}x^2 + 20x$

b. $R(15) = -\frac{1}{5}(15)^2 + 20(15)$

$= -45 + 300 = \$255$

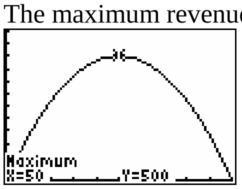
c.



d. $x = 50$ maximizes revenue

$$R(50) = -\frac{1}{5}(50)^2 + 20(50)$$
$$= -500 + 1000 = \$500$$

The maximum revenue is $500.



e. $p = \frac{100-50}{5} = \frac{50}{5} = \$10$

maximizes revenue.

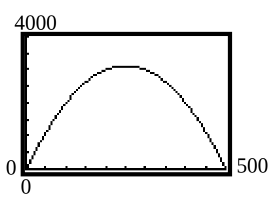
6. a. If $x = -20p + 500,$ then $p = \frac{500-x}{20}.$

$$R(x) = x\left(\frac{500-x}{20}\right) = -\frac{1}{20}x^2 + 25x$$

b. $R(20) = -\frac{1}{20}(20)^2 + 25(20)$

$$= -20 + 500 = \$480$$

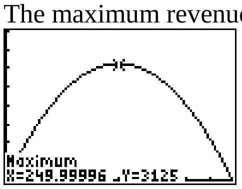
c.



d. $x = 250$ maximizes revenue

$$R(250) = -\frac{1}{20}(250)^2 + 25(250)$$
$$= -3125 + 6250 = \$3125$$

The maximum revenue is $3125.



e. $p = \frac{500-250}{20} = \frac{250}{20} = \$12.50$

maximizes revenue.

7. a. Let x = width and y = length of the rectangular area.

$P = 2x + 2y = 400$

$y = \frac{400-2x}{2} = 200 - x$

Then

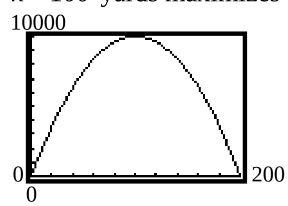
$$A(x) = (200-x)x$$
$$= 200x - x^2$$
$$= -x^2 + 200x$$

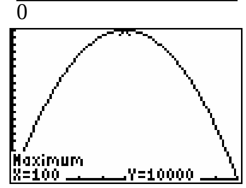
b. We need

$x > 0$ and $y > 0 \Rightarrow 200 - x > 0 \Rightarrow 200 > x$

So the domain of A is $\{x \mid 0 < x < 200\}$

c. $x = 100$ yards maximizes area





8. a. Let x = length and y = width of the rectangular field.

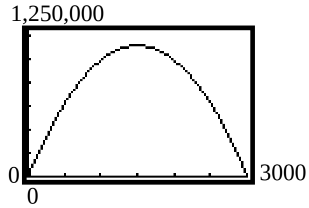
$P = x + 2y = 3000$

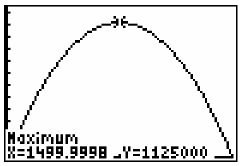
$y = \frac{3000-x}{2} = 1500 - \frac{1}{2}x$

Then,

$$A(x) = \left(1500 - \frac{1}{2}x\right)x = 1500x - \frac{1}{2}x^2$$

b. $x = 1500$ feet maximizes area





9. a. The distance d from P to the origin is $d = \sqrt{x^2 + y^2}$. Since P is a point on the graph of $y = x^2 - 8$, we have:

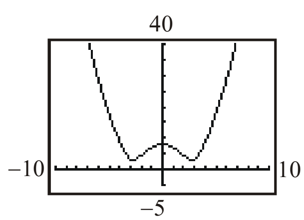
$$d(x) = \sqrt{x^2 + (x^2 - 8)^2} = \sqrt{x^4 - 15x^2 + 64}$$

b. $d(0) = \sqrt{0^4 - 15(0)^2 + 64} = \sqrt{64} = 8$

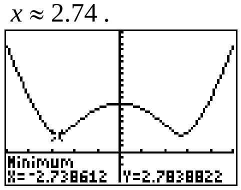
c. $d(1) = \sqrt{(1)^4 - 15(1)^2 + 64} = \sqrt{1 - 15 + 64}$

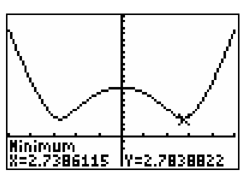
$$= \sqrt{50} = 5\sqrt{2} \approx 7.07$$

d.



e. d is smallest when $x \approx -2.74$ and when $x \approx 2.74$.





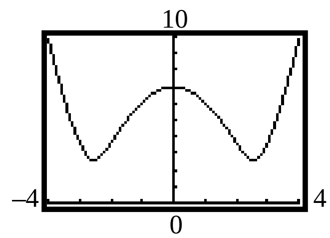
10. a. The distance d from P to (0, –1) is $d = \sqrt{x^2 + (y+1)^2}$. Since P is a point on the graph of $y = x^2 - 8$, we have:

$$d(x) = \sqrt{x^2 + (x^2 - 8 + 1)^2}$$
$$= \sqrt{x^2 + \left(x^2 - 7\right)^2}$$
$$= \sqrt{x^4 - 13x^2 + 49}$$

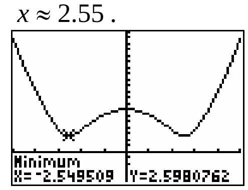
b. $d(0) = \sqrt{0^4 - 13(0)^2 + 49} = \sqrt{49} = 7$

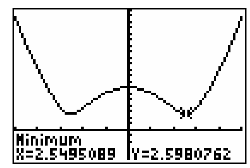
c. $d(-1) = \sqrt{(-1)^4 - 13(-1)^2 + 49} = \sqrt{37} \approx 6.08$

d.



e. d is smallest when $x \approx -2.55$ and when $x \approx 2.55$.



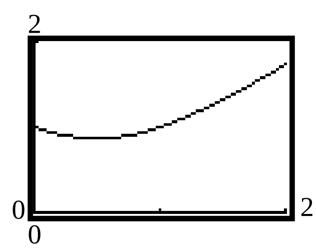


11. a. The distance d from P to the point (1, 0) is $d = \sqrt{(x-1)^2 + y^2}$. Since P is a point on the graph of $y = \sqrt{x}$, we have:

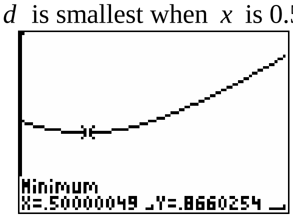
$$d(x) = \sqrt{(x-1)^2 + \left(\sqrt{x}\right)^2} = \sqrt{x^2 - x + 1}$$

where $x \geq 0$.

b.



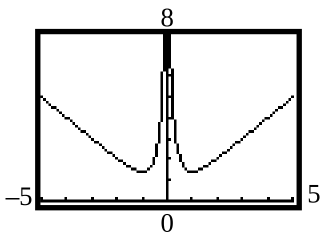
c. d is smallest when x is 0.50.



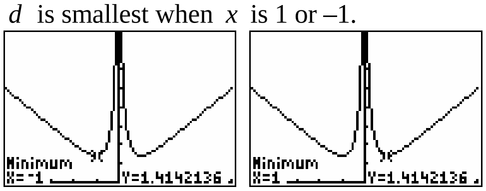
12. **a.** The distance d from P to the origin is $d=\sqrt{x^2+y^2}$. Since P is a point on the graph of $y=\dfrac{1}{x}$, we have:

$$d(x)=\sqrt{x^2+\left(\frac{1}{x}\right)^2}=\sqrt{x^2+\frac{1}{x^2}}=\sqrt{\frac{x^4+1}{x^2}}$$

b.



c. d is smallest when x is 1 or –1.



13. By definition, a triangle has area $A=\dfrac{1}{2}b\,h$, $b=$ base, $h=$ height. From the figure, we know that $b=x$ and $h=y$. Expressing the area of the triangle as a function of x, we have:

$$A(x)=\frac{1}{2}xy=\frac{1}{2}x\left(x^3\right)=\frac{1}{2}x^4.$$

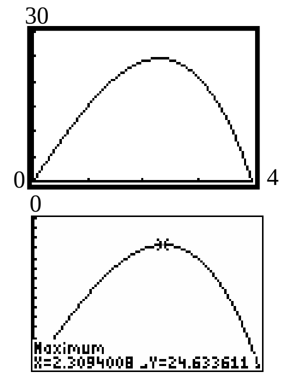
14. By definition, a triangle has area $A=\dfrac{1}{2}b\,h$, b=base, $h=$ height. Because one vertex of the triangle is at the origin and the other is on the x-axis, we know that $b=x$ and $h=y$. Expressing the area of the triangle as a function of x, we have:

$$A(x)=\frac{1}{2}xy=\frac{1}{2}x\left(9-x^2\right)=\frac{9}{2}x-\frac{1}{2}x^3.$$

15. **a.** $A(x)=xy=x\left(16-x^2\right)=-x^3+16x$

b. Domain: $\{x \mid 0<x<4\}$

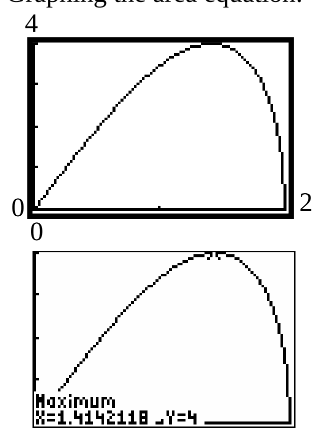
c. The area is largest when x is approximately 2.31.



16. **a.** $A(x)=2xy=2x\sqrt{4-x^2}$

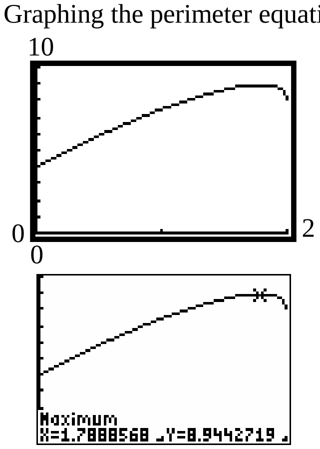
b. $p(x)=2(2x)+2(y)=4x+2\sqrt{4-x^2}$

c. Graphing the area equation:



The area is largest when x is approximately 1.41.

d. Graphing the perimeter equation:



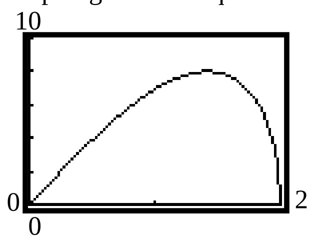
The perimeter is largest when x is approximately 1.79.

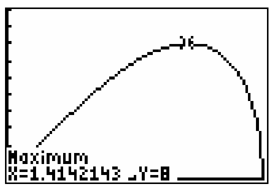
17. a. In Quadrant I, $x^2 + y^2 = 4 \rightarrow y = \sqrt{4 - x^2}$

$A(x) = (2x)(2y) = 4x\sqrt{4 - x^2}$

b. $p(x) = 2(2x) + 2(2y) = 4x + 4\sqrt{4 - x^2}$

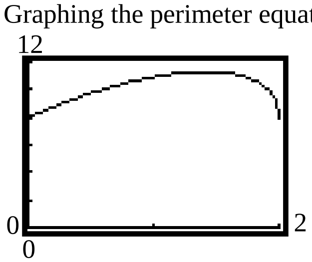
c. Graphing the area equation:

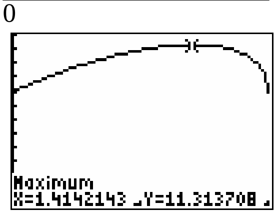




The area is largest when x is roughly 1.41.

d. Graphing the perimeter equation:





The perimeter is largest when x is approximately 1.41.

18. a. $A(r) = (2r)(2r) = 4r^2$

b. $p(r) = 4(2r) = 8r$

19. a. $C =$ circumference, $TA =$ total area, $r =$ radius, $x =$ side of square

$$C = 2\pi r = 10 - 4x \quad \Rightarrow \quad r = \frac{5 - 2x}{\pi}$$

$$\text{Total Area} = \text{area}_{\text{square}} + \text{area}_{\text{circle}}$$
$$= x^2 + \pi r^2$$
$$TA(x) = x^2 + \pi\left(\frac{5-2x}{\pi}\right)^2$$
$$= x^2 + \frac{25 - 20x + 4x^2}{\pi}$$

b. Since the lengths must be positive, we have:

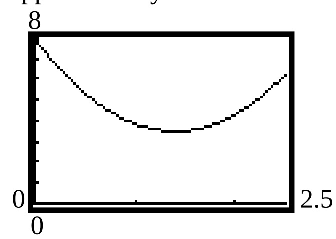
$10 - 4x > 0$ and $x > 0$

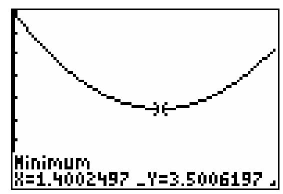
$-4x > -10$ and $x > 0$

$x < 2.5$ and $x > 0$

Domain: $\{x \mid 0 < x < 2.5\}$

c. The total area is smallest when x is approximately 1.40 meters.





20. a. $C =$ circumference, $TA =$ total area, $r =$ radius, $x =$ side of equilateral triangle

$$C = 2\pi r = 10 - 3x \Rightarrow r = \frac{10 - 3x}{2\pi}$$

height of the equilateral triangle is $\frac{\sqrt{3}}{2}x$

$$\text{Total Area} = \text{area}_{\text{triangle}} + \text{area}_{\text{circle}}$$
$$= \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right) + \pi r^2$$
$$TA(x) = \frac{\sqrt{3}}{4}x^2 + \pi\left(\frac{10 - 3x}{2\pi}\right)^2 =$$
$$\frac{\sqrt{3}}{4}x^2 + \frac{100 - 60x + 9x^2}{4\pi}$$

b. Since the lengths must be positive, we have:

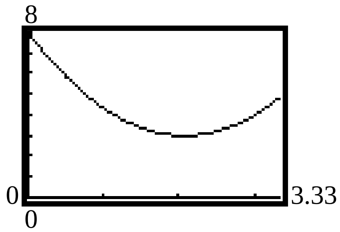
$10 - 3x > 0$ and $x > 0$

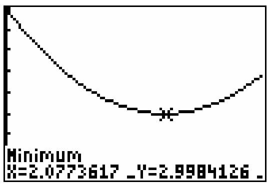
$-3x > -10$ and $x > 0$

$x < \frac{10}{3}$ and $x > 0$

Domain: $\left\{x \,\middle|\, 0 < x < \frac{10}{3}\right\}$

c. The area is smallest when x is approximately 2.08 meters.





21. a. Since the wire of length x is bent into a circle, the circumference is x. Therefore, $C(x) = x$.

b. Since $C = x = 2\pi r$, $r = \dfrac{x}{2\pi}$.

$$A(x) = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}.$$

22. a. Since the wire of length x is bent into a square, the perimeter is x. Therefore, $P(x) = x$.

b. Since $P = x = 4s$, $s = \dfrac{x}{4}$, we have

$$A(x) = s^2 = \left(\frac{x}{4}\right)^2 = \frac{x^2}{16}.$$

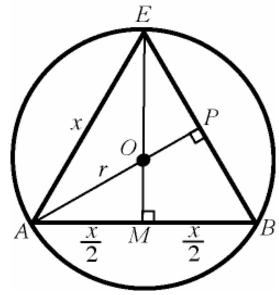
23. a. A = area, r = radius; diameter = $2r$

$$A(r) = (2r)(r) = 2r^2$$

b. p = perimeter

$$p(r) = 2(2r) + 2r = 6r$$

24. C = circumference, r = radius;
x = length of a side of the triangle



Since ΔABC is equilateral, $EM = \dfrac{\sqrt{3}x}{2}$.

Therefore,

$$OM = \frac{\sqrt{3}x}{2} - OE$$
$$= \frac{\sqrt{3}x}{2} - r$$

In ΔOAM, $r^2 = \left(\dfrac{x}{2}\right)^2 + \left(\dfrac{\sqrt{3}x}{2} - r\right)^2$

$$r^2 = \frac{x^2}{4} + \frac{3}{4}x^2 - \sqrt{3}\,rx + r^2$$
$$\sqrt{3}\,rx = x^2$$
$$r = \frac{x}{\sqrt{3}}$$

Therefore, the circumference of the circle is

$$C(x) = 2\pi r = 2\pi\left(\frac{x}{\sqrt{3}}\right) = \frac{2\pi\sqrt{3}}{3}x$$

25. Area of the equilateral triangle

$$A = \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$$

From problem 24, we have $r^2 = \dfrac{x^2}{3}$.

Area inside the circle, but outside the triangle:

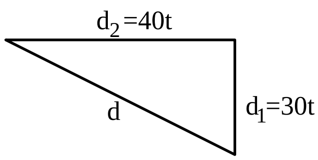
$$A(x) = \pi r^2 - \frac{\sqrt{3}}{4}x^2$$
$$= \pi\frac{x^2}{3} - \frac{\sqrt{3}}{4}x^2$$
$$= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)x^2$$

26. $d^2 = d_1^{\,2} + d_2^{\,2}$

$d^2 = (30t)^2 + (40t)^2$

$d(t) = \sqrt{900t^2 + 1600t^2}$

$d(t) = \sqrt{2500t^2} = 50t$



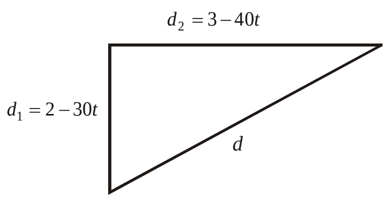
27. a. $d^2 = d_1^{\,2} + d_2^{\,2}$

$d^2 = (2-30t)^2 + (3-40t)^2$

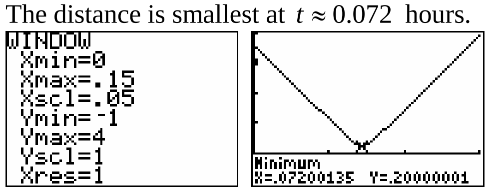
$d(t) = \sqrt{(2-30t)^2 + (3-40t)^2}$

$= \sqrt{4 - 120t + 900t^2 + 9 - 240t + 1600t^2}$

$= \sqrt{2500t^2 - 360t + 13}$



b. The distance is smallest at $t \approx 0.072$ hours.



28. r = radius of cylinder, h = height of cylinder, V = volume of cylinder

$$r^2 + \left(\frac{h}{2}\right)^2 = R^2 \Rightarrow r^2 + \frac{h^2}{4} = R^2$$

$$r^2 = R^2 - \frac{h^2}{4} \Rightarrow r^2 = \frac{4R^2 - h^2}{4}$$

$$V = \pi r^2 h$$

$$V(h) = \pi\left(\frac{4R^2 - h^2}{4}\right)h$$

$$= \frac{\pi}{4}\left(4R^2 h - h^3\right)$$

$$= \frac{\pi h}{4}\left(4R^2 - h^2\right)$$

29. r = radius of cylinder, h = height of cylinder, V = volume of cylinder

By similar triangles: $\frac{H}{R} = \frac{H-h}{r}$

$$Hr = R(H - h)$$

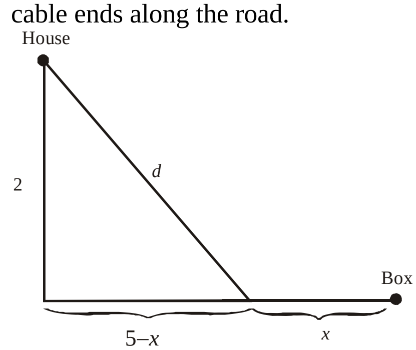
$$Hr = RH - Rh$$

$$Rh = RH - Hr$$

$$h = \frac{RH - Hr}{R} = H - \frac{Hr}{R}$$

$$V = \pi r^2 h = \pi r^2\left(H - \frac{Hr}{R}\right) = H\pi r^2\left(1 - \frac{r}{R}\right)$$

30. a. The total cost of installing the cable along the road is $10x$. If cable is installed x miles along the road, there are $5 - x$ miles between the road to the house and where the cable ends along the road.



$d = \sqrt{(5-x)^2 + 2^2} = \sqrt{25 - 10x + x^2 + 4}$

$= \sqrt{x^2 - 10x + 29}$

The total cost of installing the cable is:

$C(x) = 10x + 14\sqrt{x^2 - 10x + 29}$

Domain: $\{x \mid 0 \le x \le 5\}$

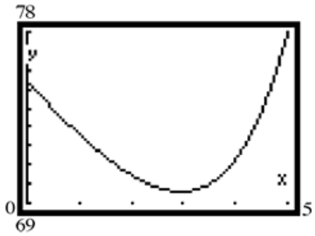
b. $C(1) = 10(1) + 14\sqrt{1^2 - 10(1) + 29}$

$= 10 + 14\sqrt{20} \approx \$72.61$

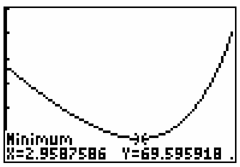
c. $C(3) = 10(3) + 14\sqrt{3^2 - 10(3) + 29}$

$= 30 + 14\sqrt{8} \approx \$69.60$

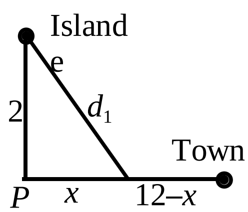
d.



e. Using MINIMUM, the graph indicates that $x \approx 2.96$ miles results in the least cost.



31. a. The time on the boat is given by $\frac{d_1}{3}$. The time on land is given by $\frac{12-x}{5}$.



$d_1 = \sqrt{x^2 + 2^2} = \sqrt{x^2 + 4}$

The total time for the trip is:

$$T(x) = \frac{12-x}{5} + \frac{d_1}{3} = \frac{12-x}{5} + \frac{\sqrt{x^2+4}}{3}$$

b. Domain: $\{x \mid 0 \le x \le 12\}$

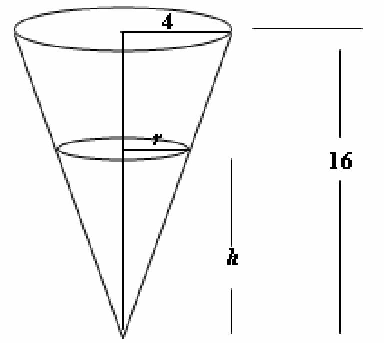
c. $T(4) = \frac{12-4}{5} + \frac{\sqrt{4^2+4}}{3}$

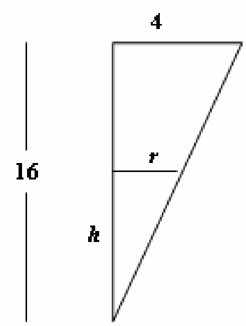
$= \frac{8}{5} + \frac{\sqrt{20}}{3} \approx 3.09$ hours

d. $T(8) = \frac{12-8}{5} + \frac{\sqrt{8^2+4}}{3}$

$= \frac{4}{5} + \frac{\sqrt{68}}{3} \approx 3.55$ hours

32. Consider the diagrams shown below.





There is a pair of similar triangles in the diagram. Since the smaller triangle is similar to the larger triangle, we have the proportion

$\frac{r}{h} = \frac{4}{16} \Rightarrow \frac{r}{h} = \frac{1}{4} \Rightarrow r = \frac{1}{4}h$

Substituting into the volume formula for the conical portion of water gives

$$V(h) = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{4}h\right)^2 h = \frac{1}{48}\pi h^3.$$

Chapter 2 Review

1. This relation represents a function.
Domain = {–1, 2, 4}; Range = {0, 3}.

2. This relation does not represent a function, since 4 is paired with two different values.

3. $f(x) = \frac{3x}{x^2 - 1}$

a. $f(2) = \frac{3(2)}{(2)^2 - 1} = \frac{6}{4-1} = \frac{6}{3} = 2$

b. $f(-2) = \frac{3(-2)}{(-2)^2 - 1} = \frac{-6}{4-1} = \frac{-6}{3} = -2$

c. $f(-x)=\dfrac{3(-x)}{(-x)^2-1}=\dfrac{-3x}{x^2-1}$

d. $-f(x)=-\left(\dfrac{3x}{x^2-1}\right)=\dfrac{-3x}{x^2-1}$

e. $f(x-2)=\dfrac{3(x-2)}{(x-2)^2-1}$
$=\dfrac{3x-6}{x^2-4x+4-1}$
$=\dfrac{3(x-2)}{x^2-4x+3}$

f. $f(2x)=\dfrac{3(2x)}{(2x)^2-1}=\dfrac{6x}{4x^2-1}$

4. $f(x)=\dfrac{x^2}{x+1}$

a. $f(2)=\dfrac{2^2}{2+1}=\dfrac{4}{3}$

b. $f(-2)=\dfrac{(-2)^2}{-2+1}=\dfrac{4}{-1}=-4$

c. $f(-x)=\dfrac{(-x)^2}{-x+1}=\dfrac{x^2}{-x+1}$

d. $-f(x)=-\dfrac{x^2}{x+1}=\dfrac{-x^2}{x+1}$

e. $f(x-2)=\dfrac{(x-2)^2}{(x-2)+1}=\dfrac{(x-2)^2}{x-1}$

f. $f(2x)=\dfrac{(2x)^2}{(2x)+1}=\dfrac{4x^2}{2x+1}$

5. $f(x)=\sqrt{x^2-4}$

a. $f(2)=\sqrt{2^2-4}=\sqrt{4-4}=\sqrt{0}=0$

b. $f(-2)=\sqrt{(-2)^2-4}=\sqrt{4-4}=\sqrt{0}=0$

c. $f(-x)=\sqrt{(-x)^2-4}=\sqrt{x^2-4}$

d. $-f(x)=-\sqrt{x^2-4}$

e. $f(x-2)=\sqrt{(x-2)^2-4}$
$=\sqrt{x^2-4x+4-4}$
$=\sqrt{x^2-4x}$

f. $f(2x)=\sqrt{(2x)^2-4}=\sqrt{4x^2-4}$
$=\sqrt{4(x^2-1)}=2\sqrt{x^2-1}$

6. $f(x)=\left|x^2-4\right|$

a. $f(2)=\left|2^2-4\right|=|4-4|=|0|=0$

b. $f(-2)=\left|(-2)^2-4\right|=|4-4|=|0|=0$

c. $f(-x)=\left|(-x)^2-4\right|=\left|x^2-4\right|$

d. $-f(x)=-\left|x^2-4\right|$

e. $f(x-2)=\left|(x-2)^2-4\right|$
$=\left|x^2-4x+4-4\right|$
$=\left|x^2-4x\right|$

f. $f(2x)=\left|(2x)^2-4\right|=\left|4x^2-4\right|$
$=\left|4(x^2-1)\right|=4\left|x^2-1\right|$

7. $f(x)=\dfrac{x^2-4}{x^2}$

a. $f(2)=\dfrac{2^2-4}{2^2}=\dfrac{4-4}{4}=\dfrac{0}{4}=0$

b. $f(-2)=\dfrac{(-2)^2-4}{(-2)^2}=\dfrac{4-4}{4}=\dfrac{0}{4}=0$

c. $f(-x)=\dfrac{(-x)^2-4}{(-x)^2}=\dfrac{x^2-4}{x^2}$

d. $-f(x)=-\left(\dfrac{x^2-4}{x^2}\right)=\dfrac{4-x^2}{x^2}=-\dfrac{x^2-4}{x^2}$

e. $f(x-2)=\dfrac{(x-2)^2-4}{(x-2)^2}=\dfrac{x^2-4x+4-4}{(x-2)^2}$
$=\dfrac{x^2-4x}{(x-2)^2}=\dfrac{x(x-4)}{(x-2)^2}$

f. $f(2x) = \dfrac{(2x)^2 - 4}{(2x)^2} = \dfrac{4x^2 - 4}{4x^2}$

$= \dfrac{4\left(x^2 - 1\right)}{4x^2} = \dfrac{x^2 - 1}{x^2}$

8. $f(x) = \dfrac{x^3}{x^2 - 9}$

a. $f(2) = \dfrac{2^3}{2^2 - 9} = \dfrac{8}{4 - 9} = \dfrac{8}{-5} = -\dfrac{8}{5}$

b. $f(2) = \dfrac{(-2)^3}{(-2)^2 - 9} = \dfrac{-8}{4 - 9} = \dfrac{-8}{-5} = \dfrac{8}{5}$

c. $f(-x) = \dfrac{(-x)^3}{(-x)^2 - 9} = \dfrac{-x^3}{x^2 - 9}$

d. $-f(x) = -\dfrac{x^3}{x^2 - 9} = \dfrac{-x^3}{x^2 - 9}$

e. $f(x-2) = \dfrac{(x-2)^3}{(x-2)^2 - 9}$

$= \dfrac{(x-2)^3}{x^2 - 4x + 4 - 9}$

$= \dfrac{(x-2)^3}{x^2 - 4x - 5}$

f. $f(2x) = \dfrac{(2x)^3}{(2x)^2 - 9} = \dfrac{8x^3}{4x^2 - 9}$

9. $f(x) = \dfrac{x}{x^2 - 9}$

The denominator cannot be zero:

$x^2 - 9 \neq 0$

$(x+3)(x-3) \neq 0$

$x \neq -3 \text{ or } 3$

Domain: $\{x \mid x \neq -3, x \neq 3\}$

10. $f(x) = \dfrac{3x^2}{x - 2}$

The denominator cannot be zero:

$x - 2 \neq 0$

$x \neq 2$

Domain: $\{x \mid x \neq 2\}$

11. $f(x) = \sqrt{2 - x}$

The radicand must be non-negative:

$2 - x \geq 0$

$x \leq 2$

Domain: $\{x \mid x \leq 2\}$ or $(-\infty, 2]$

12. $f(x) = \sqrt{x + 2}$

The radicand must be non-negative:

$x + 2 \geq 0$

$x \geq -2$

Domain: $\{x \mid x \geq -2\}$ or $[-2, \infty)$

13. $f(x) = \dfrac{\sqrt{x}}{|x|}$

The radicand must be non-negative and the denominator cannot be zero: $x > 0$

Domain: $\{x \mid x > 0\}$ or $(0, \infty)$

14. $g(x) = \dfrac{|x|}{x}$

The denominator cannot be zero:

$x \neq 0$

Domain: $\{x \mid x \neq 0\}$

15. $f(x) = \dfrac{x}{x^2 + 2x - 3}$

The denominator cannot be zero:

$x^2 + 2x - 3 \neq 0$

$(x+3)(x-1) \neq 0$

$x \neq -3 \text{ or } 1$

Domain: $\{x \mid x \neq -3,\ x \neq 1\}$

16. $F(x) = \dfrac{1}{x^2 - 3x - 4}$

The denominator cannot be zero:

$x^2 - 3x - 4 \neq 0$

$(x+1)(x-4) \neq 0$

$x \neq -1 \text{ or } 4$

Domain: $\{x \mid x \neq -1,\ x \neq 4\}$

17. $f(x)=2-x \quad g(x)=3x+1$

$(f+g)(x)=f(x)+g(x)$
$=2-x+3x+1=2x+3$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f-g)(x)=f(x)-g(x)$
$=2-x-(3x+1)$
$=2-x-3x-1$
$=-4x+1$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f\cdot g)(x)=f(x)\cdot g(x)$
$=(2-x)(3x+1)$
$=6x+2-3x^2-x$
$=-3x^2+5x+2$

Domain: $\{x \mid x \text{ is any real number}\}$

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{2-x}{3x+1}$

$3x+1\neq 0$

$3x\neq -1 \Rightarrow x\neq -\frac{1}{3}$

Domain: $\left\{x \,\middle|\, x\neq -\frac{1}{3}\right\}$

18. $f(x)=2x-1 \qquad g(x)=2x+1$

$(f+g)(x)=f(x)+g(x)$
$=2x-1+2x+1$
$=4x$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f-g)(x)=f(x)-g(x)$
$=2x-1-(2x+1)$
$=2x-1-2x-1$
$=-2$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f\cdot g)(x)=f(x)\cdot g(x)$
$=(2x-1)(2x+1)$
$=4x^2+2x-2x-1$
$=4x^2-1$

Domain: $\{x \mid x \text{ is any real number}\}$

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{2x-1}{2x+1}$

$2x+1\neq 0 \Rightarrow 2x\neq -1 \Rightarrow x\neq -\frac{1}{2}$

Domain: $\left\{x \,\middle|\, x\neq -\frac{1}{2}\right\}$

19. $f(x)=3x^2+x+1 \qquad g(x)=3x$

$(f+g)(x)=f(x)+g(x)$
$=3x^2+x+1+3x$
$=3x^2+4x+1$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f-g)(x)=f(x)-g(x)$
$=3x^2+x+1-3x$
$=3x^2-2x+1$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f\cdot g)(x)=f(x)\cdot g(x)$
$=(3x^2+x+1)(3x)$
$=9x^3+3x^2+3x$

Domain: $\{x \mid x \text{ is any real number}\}$

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{3x^2+x+1}{3x}$

$3x\neq 0 \Rightarrow x\neq 0$

Domain: $\{x \mid x\neq 0\}$

20. $f(x)=3x \qquad g(x)=1+x+x^2$

$(f+g)(x)=f(x)+g(x)$
$=3x+1+x+x^2$
$=x^2+4x+1$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f-g)(x)=f(x)-g(x)$
$=3x-(1+x+x^2)$
$=-x^2+2x-1$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f\cdot g)(x)=f(x)\cdot g(x)$
$=(3x)(1+x+x^2)$
$=3x+3x^2+3x^3$

Domain: $\{x \mid x \text{ is any real number}\}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x}{1+x+x^2}$

$1+x+x^2 \neq 0$

$x^2+x+1 \neq 0$

Since the discriminant is $1^2-4(1)(1)=-3<0$,

x^2+x+1 will never equal 0.

Domain: $\{x \mid x \text{ is any real number}\}$

21. $f(x)=\frac{x+1}{x-1}$ $g(x)=\frac{1}{x}$

$(f+g)(x) = f(x)+g(x)$

$= \frac{x+1}{x-1}+\frac{1}{x}$

$= \frac{x(x+1)+1(x-1)}{x(x-1)}$

$= \frac{x^2+x+x-1}{x(x-1)}$

$= \frac{x^2+2x-1}{x(x-1)}$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

$(f-g)(x) = f(x)-g(x)$

$= \frac{x+1}{x-1}-\frac{1}{x}$

$= \frac{x(x+1)-1(x-1)}{x(x-1)}$

$= \frac{x^2+x-x+1}{x(x-1)}$

$= \frac{x^2+1}{x(x-1)}$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

$(f \cdot g)(x) = f(x)\cdot g(x) = \left(\frac{x+1}{x-1}\right)\left(\frac{1}{x}\right) = \frac{x+1}{x(x-1)}$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{x+1}{x-1}}{\frac{1}{x}} = \left(\frac{x+1}{x-1}\right)\left(\frac{x}{1}\right) = \frac{x(x+1)}{x-1}$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

22. $f(x)=\frac{1}{x-3}$ $g(x)=\frac{3}{x}$

$(f+g)(x) = f(x)+g(x)$

$= \frac{1}{x-3}+\frac{3}{x} = \frac{x+3(x-3)}{x(x-3)}$

$= \frac{x+3x-9}{x(x-3)} = \frac{4x-9}{x(x-3)}$

Domain: $\{x \mid x \neq 0, x \neq 3\}$

$(f-g)(x) = f(x)-g(x) = \frac{1}{x-3}-\frac{3}{x}$

$= \frac{x-3(x-3)}{x(x-3)} = \frac{x-3x+9}{x(x-3)}$

$= \frac{-2x+9}{x(x-3)}$

Domain: $\{x \mid x \neq 0, x \neq 3\}$

$(f \cdot g)(x) = f(x)\cdot g(x) = \left(\frac{1}{x-3}\right)\left(\frac{3}{x}\right) = \frac{3}{x(x-3)}$

Domain: $\{x \mid x \neq 0, x \neq 3\}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{1}{x-3}}{\frac{3}{x}} = \left(\frac{1}{x-3}\right)\left(\frac{x}{3}\right)$

$= \frac{x}{3(x-3)}$

Domain: $\{x \mid x \neq 0, x \neq 3\}$

23. $f(x)=-2x^2+x+1$

$\frac{f(x+h)-f(x)}{h}$

$= \frac{-2(x+h)^2+(x+h)+1-(-2x^2+x+1)}{h}$

$= \frac{-2(x^2+2xh+h^2)+x+h+1+2x^2-x-1}{h}$

$= \frac{-2x^2-4xh-2h^2+x+h+1+2x^2-x-1}{h}$

$= \frac{-4xh-2h^2+h}{h} = \frac{h(-4x-2h+1)}{h}$

$= -4x-2h+1$

24. $f(x) = 3x^2 - 2x + 4$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{3(x+h)^2 - 2(x+h) + 4 - (3x^2 - 2x + 4)}{h}$$
$$= \frac{3(x^2 + 2xh + h^2) - 2x - 2h + 4 - 3x^2 + 2x - 4}{h}$$
$$= \frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 4 - 3x^2 + 2x - 4}{h}$$
$$= \frac{6xh + 3h^2 - 2h}{h} = \frac{h(6x + 3h - 2)}{h}$$
$$= 6x + 3h - 2$$

25. a. Domain: $\{ x | -4 \le x \le 3 \}$
Range: $\{ y | -3 \le y \le 3 \}$

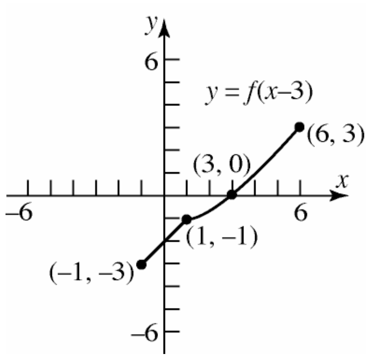
b. x-intercept: $(0,0)$; y-intercept: $(0,0)$

c. $f(-2) = -1$

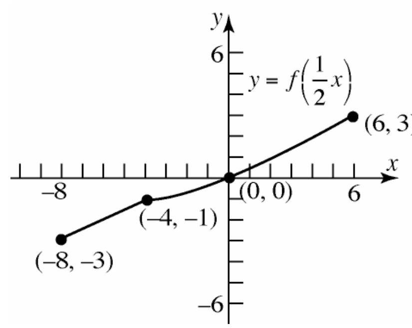
d. $f(x) = -3$ when $x = -4$

e. $f(x) > 0$ when $0 < x \le 3$

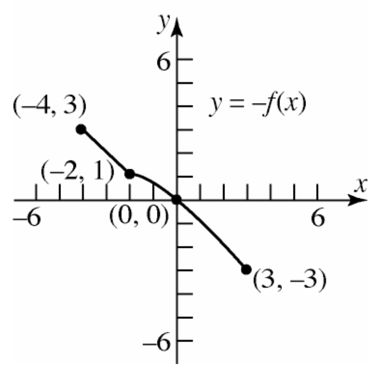
f. To graph $y = f(x-3)$, shift the graph of f horizontally 3 units to the right.



g. To graph $y = f\left(\frac{1}{2}x\right)$, stretch the graph of f horizontally by a factor of 2.



h. To graph $y = -f(x)$, reflect the graph of f vertically about the y-axis.



26. a. Domain: $\{ x | -5 \le x \le 4 \}$
Range: $\{ y | -3 \le y \le 1 \}$

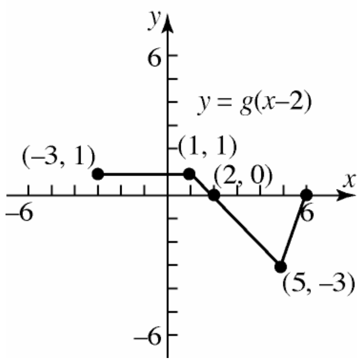
b. $g(-1) = 1$

c. x-intercepts: $(0,0)$, $(4,0)$;
y-intercept: $(0,0)$

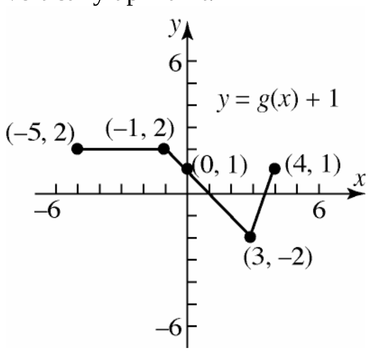
d. $g(x) = -3$ when $x = 3$

e. $g(x) > 0$ when $-5 \le x < 0$

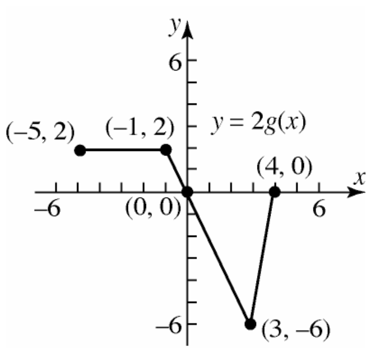
f. To graph $y = g(x-2)$, shift the graph of g horizontally 2 units to the right.



g. To graph $y = g(x)+1$, shift the graph of g vertically up 1 unit.



h. To graph $y = 2g(x)$, stretch the graph of g vertically by a factor of 2.



27. a. Domain: $\{x \mid -4 \le x \le 4\}$

Range: $\{y \mid -3 \le y \le 1\}$

b. Increasing: $(-4,-1)$ and $(3,4)$;
Decreasing: (–1, 3)

c. Local minimum is -3 when $x = 3$;
Local maximum is 1 when $x = -1$.
Note that $x = 4$ and $x = -4$ do not yield local extrema because there is no open interval that contains either value.

d. The graph is not symmetric with respect to the *x*-axis, the *y*-axis or the origin.

e. The function is neither even nor odd.

f. *x*-intercepts: $(-2,0)$, $(0,0)$, $(4,0)$,
y-intercept: $(0,0)$

28. a. Domain: $\{x \mid x \text{ is any real number}\}$
Range: $\{y \mid y \text{ is any real number}\}$

b. Increasing: $(-\infty,-2)$ and $(2,\infty)$;
Decreasing: $(-2,2)$

c. Local minimum is -1 at $x = 2$;
Local maximum is 1 at $x = -2$

d. The graph is symmetric with respect to the origin.

e. The function is odd.

f. *x*-intercepts: $(-3,0)$, $(0,0)$, $(3,0)$;
y-intercept: $(0,0)$

29. $f(x) = x^3 - 4x$

$$f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x$$
$$= -\left(x^3 - 4x\right) = -f(x)$$

f is odd.

30. $g(x) = \dfrac{4+x^2}{1+x^4}$

$$g(-x) = \frac{4+(-x)^2}{1+(-x)^4} = \frac{4+x^2}{1+x^4} = g(x)$$

g is even.

31. $h(x) = \dfrac{1}{x^4} + \dfrac{1}{x^2} + 1$

$$h(-x) = \frac{1}{(-x)^4} + \frac{1}{(-x)^2} + 1 = \frac{1}{x^4} + \frac{1}{x^2} + 1 = h(x)$$

h is even.

32. $F(x)=\sqrt{1-x^3}$

$F(-x)=\sqrt{1-(-x)^3}=\sqrt{1+x^3}\neq F(x)$ or $-F(x)$

F is neither even nor odd.

33. $G(x)=1-x+x^3$

$G(-x)=1-(-x)+(-x)^3$

$=1+x-x^3\neq -G(x)$ or $G(x)$

G is neither even nor odd.

34. $H(x)=1+x+x^2$

$H(-x)=1+(-x)+(-x)^2$

$=1-x+x^2\neq -H(x)$ or $H(x)$

H is neither even nor odd.

35. $f(x)=\dfrac{x}{1+x^2}$

$f(-x)=\dfrac{-x}{1+(-x)^2}=\dfrac{-x}{1+x^2}=-f(x)$

f is odd.

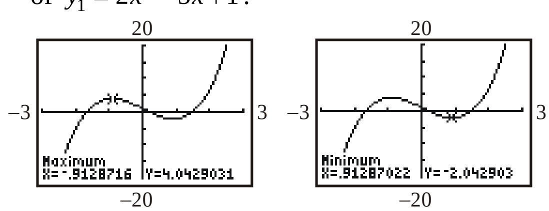
36. $g(x)=\dfrac{1+x^2}{x^3}$

$g(-x)=\dfrac{1+(-x)^2}{(-x)^3}=\dfrac{1+x^2}{-x^3}=-\dfrac{1+x^2}{x^3}=-g(x)$

g is odd.

37. $f(x)=2x^3-5x+1$ on the interval $(-3,3)$

Use MAXIMUM and MINIMUM on the graph of $y_1=2x^3-5x+1$.



local maximum at: $(-0.91,4.04)$;

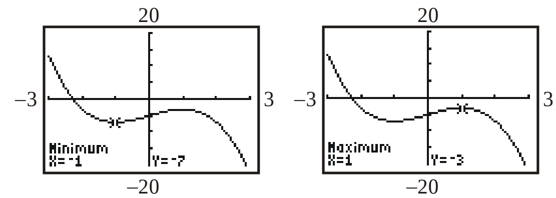
local minimum at: $(0.91,-2.04)$

f is increasing on: $(-3,-0.91)$ and $(0.91,3)$;

f is decreasing on: $(-0.91,0.91)$

38. $f(x)=-x^3+3x-5$ on the interval $(-3,3)$

Use MAXIMUM and MINIMUM on the graph of $y_1=-x^3+3x-5$.



local maximum at: $(1,-3)$;

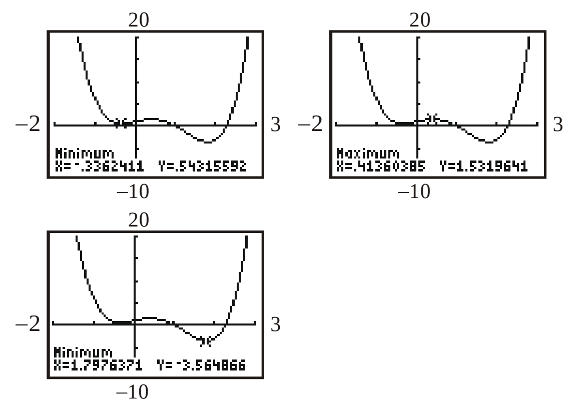
local minimum at: $(-1,-7)$

f is increasing on: $(-1,1)$;

f is decreasing on: $(-3,-1)$ and $(1,3)$

39. $f(x)=2x^4-5x^3+2x+1$ on the interval $(-2,3)$

Use MAXIMUM and MINIMUM on the graph of $y_1=2x^4-5x^3+2x+1$.



local maximum at: $(0.41,1.53)$;

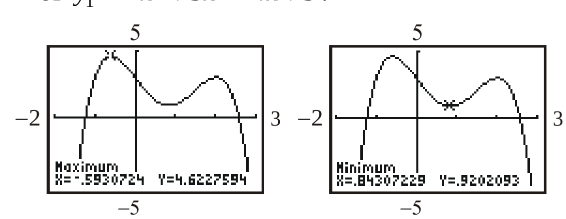
local minima at: $(-0.34,0.54)$, $(1.80,-3.56)$

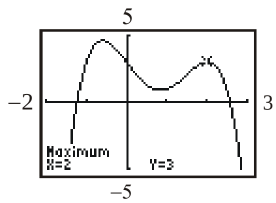
f is increasing on: $(-0.34,0.41)$ and $(1.80,3)$;

f is decreasing on: $(-2,-0.34)$ and $(0.41,1.80)$

40. $f(x)=-x^4+3x^3-4x+3$ on the interval $(-2,3)$

Use MAXIMUM and MINIMUM on the graph of $y_1=-x^4+3x^3-4x+3$.





local maxima at: $(-0.59, 4.62)$, $(2, 3)$;
local minimum at: $(0.84, 0.92)$
f is increasing on: $(-2, -0.59)$ and $(0.84, 2)$;
f is decreasing on: $(-0.59, 0.84)$ and $(2, 3)$

41. $f(x) = 8x^2 - x$

a. $$\frac{f(2)-f(1)}{2-1} = \frac{8(2)^2 - 2 - \left(8(1)^2 - 1\right)}{1} = 32 - 2 - (7) = 23$$

b. $$\frac{f(1)-f(0)}{1-0} = \frac{8(1)^2 - 1 - \left(8(0)^2 - 0\right)}{1} = 8 - 1 - (0) = 7$$

c. $$\frac{f(4)-f(2)}{4-2} = \frac{8(4)^2 - 4 - \left(8(2)^2 - 2\right)}{2} = \frac{128 - 4 - (30)}{2} = \frac{94}{2} = 47$$

42. $f(x) = 2x^3 + x$

a. $$\frac{f(2)-f(1)}{2-1} = \frac{2(2)^3 + 2 - \left(2(1)^3 + 1\right)}{1} = 16 + 2 - (3) = 15$$

b. $$\frac{f(1)-f(0)}{1-0} = \frac{2(1)^3 + 1 - \left(2(0)^3 + 0\right)}{1} = 2 + 1 - (0) = 3$$

c. $$\frac{f(4)-f(2)}{4-2} = \frac{2(4)^3 + 4 - \left(2(2)^3 + 2\right)}{2} = \frac{128 + 4 - (18)}{2} = \frac{114}{2} = 57$$

43. $f(x) = 2 - 5x$

$$\frac{f(x)-f(2)}{x-2} = \frac{2-5x-(-8)}{x-2} = \frac{-5x+10}{x-2} = \frac{-5(x-2)}{x-2} = -5$$

44. $f(x) = 2x^2 + 7$

$$\frac{f(x)-f(2)}{x-2} = \frac{2x^2+7-15}{x-2} = \frac{2x^2-8}{x-2} = \frac{2(x-2)(x+2)}{x-2} = 2x+4$$

45. $f(x) = 3x - 4x^2$

$$\frac{f(x)-f(2)}{x-2} = \frac{3x-4x^2-(-10)}{x-2} = \frac{-4x^2+3x+10}{x-2} = \frac{-(4x^2-3x-10)}{x-2} = \frac{-(4x+5)(x-2)}{x-2} = -4x-5$$

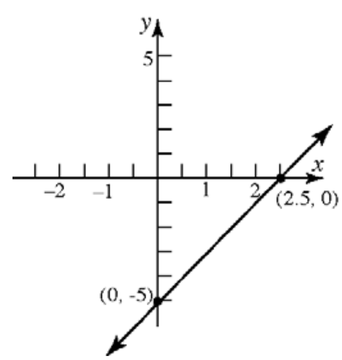
46. $f(x) = x^2 - 3x + 2$

$$\frac{f(x)-f(2)}{x-2} = \frac{x^2-3x+2-0}{x-2} = \frac{x^2-3x+2}{x-2} = \frac{(x-2)(x-1)}{x-2} = x-1$$

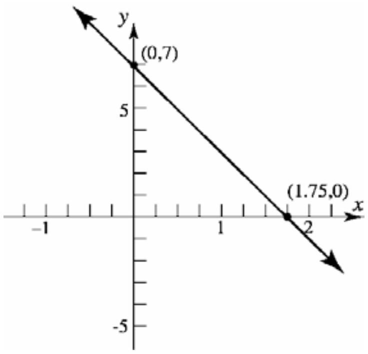
47. (b) passes the Vertical Line Test and is therefore a function.

48. (a) and (b) both pass the Vertical Line Test and are therefore functions.

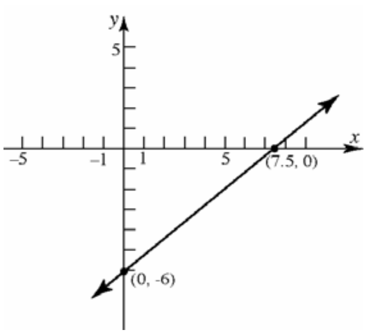
49. $f(x) = 2x - 5$



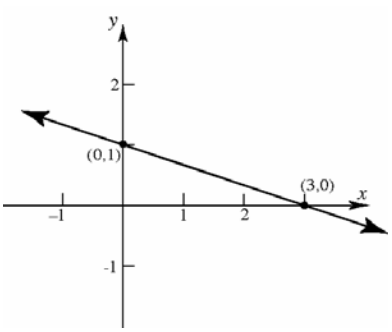
50. $g(x) = -4x + 7$



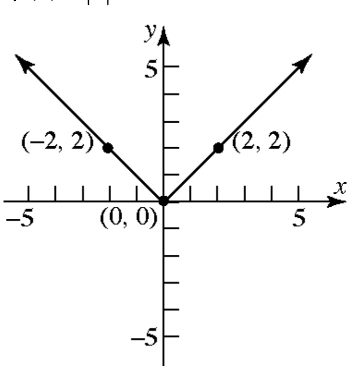
51. $h(x) = \frac{4}{5}x - 6$



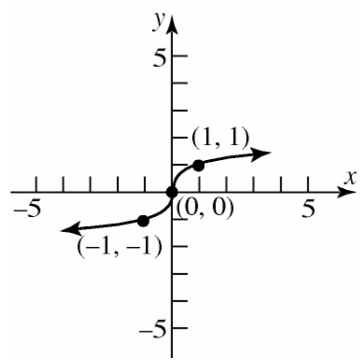
52. $F(x) = -\frac{1}{3}x + 1$



53. $f(x) = |x|$

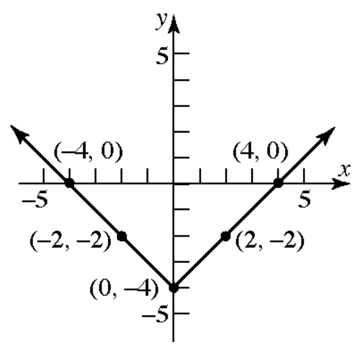


54. $f(x) = \sqrt[3]{x}$



55. $F(x) = |x| - 4$

Using the graph of $y = |x|$, vertically shift the graph downward 4 units.



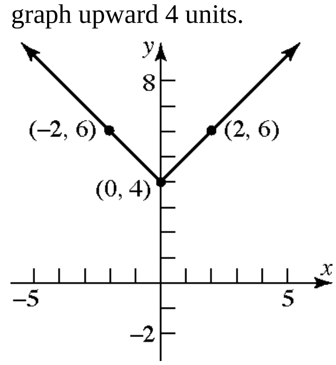
Intercepts: (–4,0), (4,0), (0,–4)

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \geq -4\}$

56. $f(x) = |x| + 4$

Using the graph of $y = |x|$, vertically shift the graph upward 4 units.



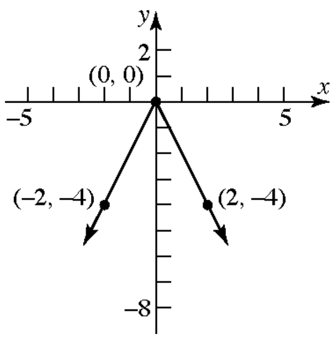
Intercepts: (0, 4)

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \geq 4\}$

57. $g(x) = -2|x|$

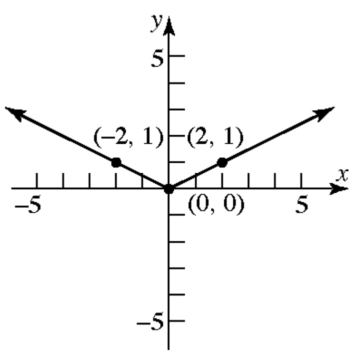
Reflect the graph of $y = |x|$ about the *x*-axis and vertically stretch the graph by a factor of 2.



Intercepts: (0, 0)
Domain: $\{x | x \text{ is any real number}\}$
Range: $\{y | y \le 0\}$

58. $g(x) = \frac{1}{2}|x|$

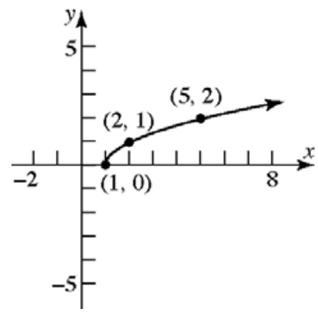
Using the graph of $y = |x|$, vertically shrink the graph by a factor of $\frac{1}{2}$.



Intercepts: (0, 0)
Domain: $\{x | x \text{ is any real number}\}$
Range: $\{y | y \ge 0\}$

59. $h(x) = \sqrt{x-1}$

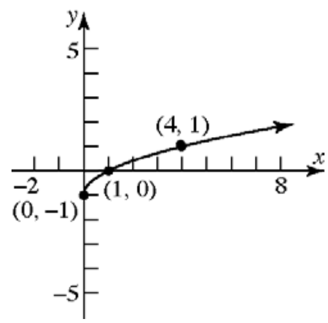
Using the graph of $y = \sqrt{x}$, horizontally shift the graph to the right 1 unit.



Intercept: (1, 0)
Domain: $\{x | x \ge 1\}$; Range: $\{y | y \ge 0\}$

60. $h(x) = \sqrt{x} - 1$

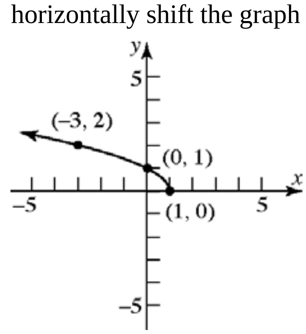
Using the graph of $y = \sqrt{x}$, vertically shift the graph downward 1 unit.



Intercepts: (1, 0), (0, –1)
Domain: $\{x | x \ge 0\}$; Range: $\{y | y \ge -1\}$

61. $f(x) = \sqrt{1-x} = \sqrt{-1(x-1)}$

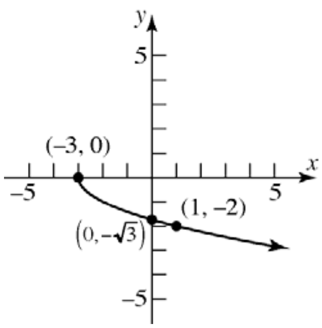
Reflect the graph of $y = \sqrt{x}$ about the *y*-axis and horizontally shift the graph to the right 1 unit.



Intercepts: (1 ,0), (0,1)
Domain: $\{x | x \le 1\}$
Range: $\{y | y \ge 0\}$

62. $f(x) = -\sqrt{x+3}$

Using the graph of $y = \sqrt{x}$, horizontally shift the graph to the left 3 units, and reflect on the x-axis.



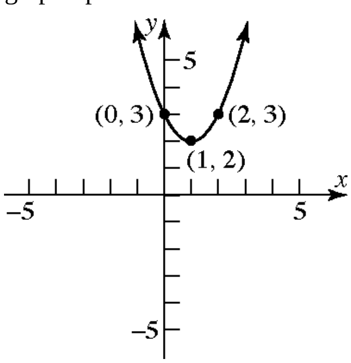
Intercepts: (–3, 0), $\left(0, -\sqrt{3}\right)$

Domain: $\{x \mid x \geq -3\}$

Range: $\{y \mid y \leq 0\}$

63. $h(x) = (x-1)^2 + 2$

Using the graph of $y = x^2$, horizontally shift the graph to the right 1 unit and vertically shift the graph up 2 units.



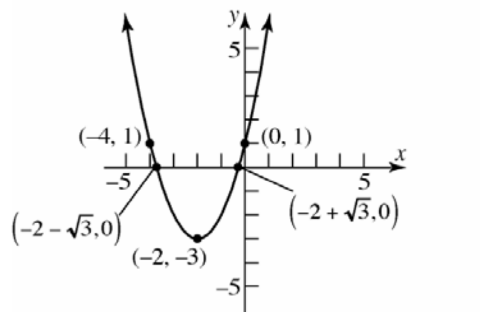
Intercepts: (0, 3)

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \geq 2\}$

64. $h(x) = (x+2)^2 - 3$

Using the graph of $y = x^2$, horizontally shift the graph to the left 2 units and vertically shift the graph down 3 units.



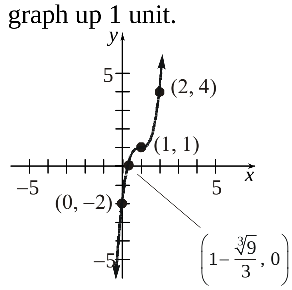
Intercepts: (0, 1), $\left(-2+\sqrt{3}, 0\right), \left(-2-\sqrt{3}, 0\right)$

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \geq -3\}$

65. $g(x) = 3(x-1)^3 + 1$

Using the graph of $y = x^3$, horizontally shift the graph to the right 1 unit vertically stretch the graph by a factor of 3, and vertically shift the graph up 1 unit.



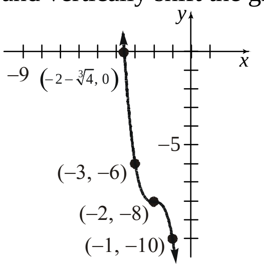
Intercepts: (0,–2), $\left(1-\frac{\sqrt[3]{9}}{3}, 0\right)$

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \text{ is any real number}\}$

66. $g(x) = -2(x+2)^3 - 8$

Using the graph of $y = x^3$, horizontally shift the graph to the left 2 units, vertically stretch the graph by a factor of 2, reflect about the x-axis, and vertically shift the graph down 8 units.



Intercepts: (0,–24), $\left(-2-\sqrt[3]{4},\ 0\right)$

Domain: $\{x \mid x \text{ is any real number}\}$

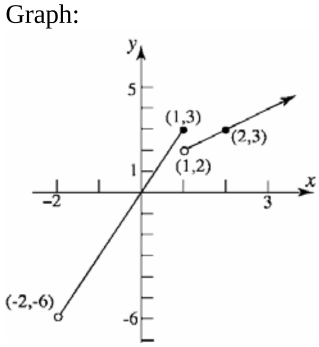
Range: $\{y \mid y \text{ is any real number}\}$

67. $f(x) = \begin{cases} 3x & \text{if } -2 < x \le 1 \\ x+1 & \text{if } x > 1 \end{cases}$

a. Domain: $\{x \mid x > -2\}$

b. x-intercept: $(0,0)$
y-intercept: (0,0)

c. Graph:



d. Range: $\{y > -6\}$

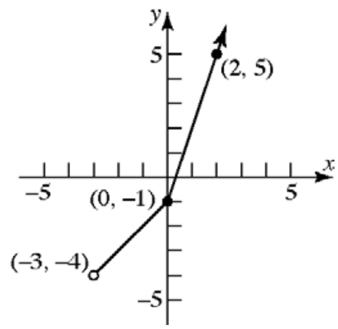
68. $f(x) = \begin{cases} x-1 & \text{if } -3 < x < 0 \\ 3x-1 & \text{if } x \ge 0 \end{cases}$

a. Domain: $\{x \mid x > -3\}$

b. x-intercept: $\left(\dfrac{1}{3}, 0\right)$

y-intercept: (0, –1)

c. Graph:



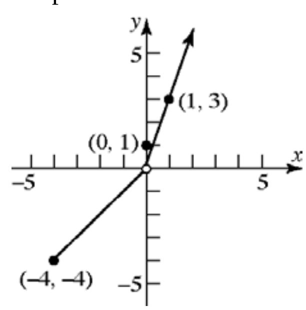
d. Range: $\{y > -4\}$

69. $f(x) = \begin{cases} x & \text{if } -4 \le x < 0 \\ 1 & \text{if } x = 0 \\ 3x & \text{if } x > 0 \end{cases}$

a. Domain: $\{x \mid x \ge -4\}$

b. x-intercept: none
y-intercept: (0, 1)

c. Graph:



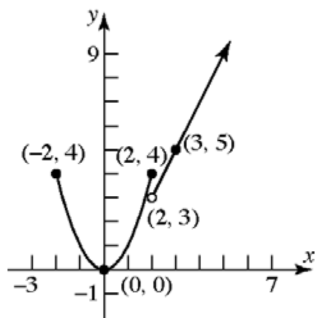
d. Range: $\{y \mid y \ge -4,\ y \ne 0\}$

70. $f(x) = \begin{cases} x^2 & \text{if } -2 \le x \le 2 \\ 2x-1 & \text{if } x > 2 \end{cases}$

a. Domain: $\{x \mid x \ge -2\}$

b. x-intercept: (0, 0)
y-intercept: (0, 0)

c. Graph:



d. Range: $\{y \mid y \geq 0\}$

71. $f(4) = -5$ gives the ordered pair $(4, -5)$

$f(0) = 3$ gives $f(0) = 3$ gives $(0, 3)$

Finding the slope: $m = \dfrac{3-(-5)}{0-4} = \dfrac{8}{-4} = -2$

Using slope-intercept form: $f(x) = -2x + 3$

72. $m = -4$, $g(-2) = 2$ gives the ordered pair $(-2, 2)$.

Using point-slope form:

$y - 2 = -4(x - (-2))$

$y - 2 = -4x - 8$

$y = -4x - 6$

$g(x) = -4x - 6$

73. $f(x) = \dfrac{Ax+5}{6x-2}$ and $f(1) = 4$

$\dfrac{A(1)+5}{6(1)-2} = 4$

$\dfrac{A+5}{4} = 4$

$A + 5 = 16$

$A = 11$

74. $g(x) = \dfrac{A}{x} + \dfrac{8}{x^2}$ and $g(-1) = 0$

$\dfrac{A}{-1} + \dfrac{8}{(-1)^2} = 0$

$-A + 8 = 0$

$A = 8$

75. We have the points $(h_1, T_1) = (0, 30)$ and $(h_2, T_2) = (10000, 5)$.

slope $= \dfrac{\Delta T}{\Delta h} = \dfrac{5-30}{10000-0} = \dfrac{-25}{10000} = -0.0025$

Using the point-slope formula yields

$T - T_1 = m(h - h_1) \Rightarrow T - 30 = -0.0025(h - 0)$

$T - 30 = -0.0025h \Rightarrow T = -0.0025h + 30$

$T(h) = -0.0025h + 30, \quad 0 \leq x \leq 10{,}000$

76. We have the point $(t_1, v_1) = (20, 80)$ and

slope $= m = 5$

Using the point-slope formula yields

$v - v_1 = m(t - t_1) \Rightarrow v - 80 = 5(t - 20)$

$v - 80 = 5t - 100 \Rightarrow v = 5t - 20$

$v(t) = 5t - 20$

After 30 seconds,

$v(30) = 5(30) - 20 = 150 - 20 = 130$ ft. per sec.

77. $S = 4\pi r^2$; $V = \dfrac{4}{3}\pi r^3$

Let $R = 2r$, $S_2 =$ new surface area, and $V_2 =$ new volume.

$S_2 = 4\pi R^2 = 4\pi(2r)^2$

$= 4\pi(4r^2) = 4(4\pi r^2)$

$= 4S$

$V_2 = \dfrac{4}{3}\pi R^3 = \dfrac{4}{3}\pi(2r)^3$

$= \dfrac{4}{3}\pi(8r^3) = 8\left(\dfrac{4}{3}\pi r^3\right)$

$= 8V$

Thus, if the radius of the sphere doubles, the surface area is 4 times as large and the volume is 8 times as large as for the original sphere.

78. a. The printed region is a rectangle. Its area is given by

$A = (\text{length})(\text{width}) = (11 - 2x)(8.5 - 2x)$

$A(x) = (11 - 2x)(8.5 - 2x)$

b. For the domain of

$A(x) = (11-2x)(8.5-2x)$

recall that the dimensions of a rectangle must be non-negative.

$x \geq 0$ and $11-2x \geq 0$ and $8.5-2x \geq 0$

$-2x \geq -11$ $\quad$ $-2x \geq 8.5$

$x \leq 5.5$ $\quad$ $x \leq 4.25$

The domain is $\{x \mid 0 \leq x \leq 4.25\}$.

The range of $A(x) = (11-2x)(8.5-2x)$ is given by

$A(4.25) \leq A \leq A(0) \Rightarrow 0 \leq A \leq 93.5$

c. $A(1) = (11-2(1))(8.5-2(1))$

$= 9 \cdot 6.5 = 58.5 \text{ in}^2$

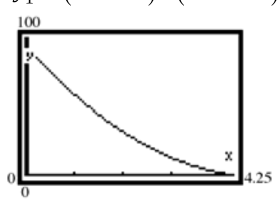
$A(1.2) = (11-2(1.2))(8.5-2(1.2))$

$= 8.6 \cdot 6.1 = 52.46 \text{ in}^2$

$A(1.5) = (11-2(1.5))(8.5-2(1.5))$

$= 8 \cdot 5.5 = 44 \text{ in}^2$

d. $y_1 = (11-2x)*(8.5-2x)$



e. Using TRACE,

$A \approx 70$ when $x \approx 0.643$ inches

$A = 50$ when $x \approx 1.28$ inches

79. $S = kxd^3$, x = width; d = depth

In the diagram, depth = length of the rectangle. Therefore, we have

$$\left(\frac{d}{2}\right)^2 + \left(\frac{x}{2}\right)^2 = 3^2$$

$$\frac{d^2}{4} + \frac{x^2}{4} = 9$$

$$d^2 + x^2 = 36$$

$$d = \sqrt{36-x^2}$$

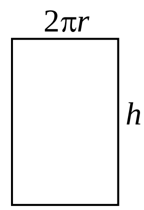
$$S(x) = kx\left(\sqrt{36-x^2}\right)^3 = kx\left(36-x^2\right)^{3/2}$$

Domain: $\{x \mid 0 < x < 6\}$

80. a. We are given that the volume is 100 cubic feet, so we have

$$V = \pi r^2 h = 100 \Rightarrow h = \frac{100}{\pi r^2}$$

The amount of material needed to construct the drum is the surface area of the barrel. The cylindrical body of the barrel can be viewed as a rectangle whose dimensions are given by



$A = \text{area}_{\text{top}} + \text{area}_{\text{bottom}} + \text{area}_{\text{body}}$

$= \pi r^2 + \pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi rh$

$$A(r) = 2\pi r^2 + 2\pi r\left(\frac{100}{\pi r^2}\right) = 2\pi r^2 + \frac{200}{r}$$

b. $A(3) = 2\pi(3)^2 + \dfrac{200}{3}$

$= 18\pi + \dfrac{200}{3} \approx 123.22 \text{ ft}^2$

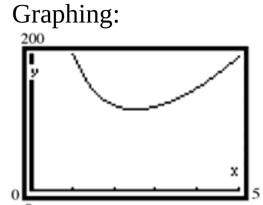
c. $A(4) = 2\pi(4)^2 + \dfrac{200}{4}$

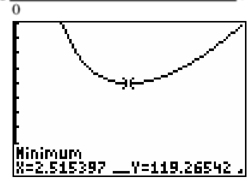
$= 32\pi + 50 \approx 150.53 \text{ ft}^2$

d. $A(5) = 2\pi(5)^2 + \dfrac{200}{5}$

$= 50\pi + 40 \approx 197.08 \text{ ft}^2$

e. Graphing:

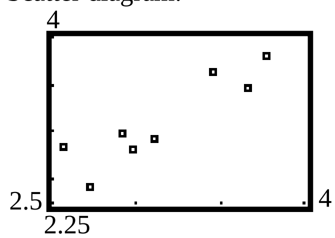




The minimum value occurs when $r \approx 2.52$ feet.

81. **a.** The relation is a function. Each HS GPA value is paired with exactly one College GPA value.

b. Scatter diagram:



c. Using the LINear REGression program, the line of best fit is: $G = 0.964x + 0.072$

d. As the high school GPA increases by 0.1 point, the college GPA increases by 0.0964 point.

e. $G(x) = 0.964x + 0.072$

f. Domain: $\{x \mid 0 \le x \le 4\}$

g. $G(3.23) = (0.964)(3.23) + 0.072 \approx 3.19$

The college GPA is approximately 3.19.

82. Let $p =$ the monthly payment in dollars, and B = the amount borrowed in dollars. Consider the ordered pair (B, p). We can use the points $(0,0)$ and $(130000, 854)$. Now compute the slope:

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{854 - 0}{130000 - 0} = \frac{854}{130000} \approx 0.0065692$$

Therefore we have the linear function

$p(B) = 0.0065692B + 0 = 0.0065692B$

If $B = 165000$, then

$p = (0.0065692)(165000) \approx \$1083.92$.

83. Let R = the revenue in dollars, and g = the number of gallons of gasoline sold. Consider the ordered pair (g, R). We can use the points $(0,0)$ and $(13.5, 28.89)$. Now compute the slope:

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{28.89 - 0}{13.5 - 0} = \frac{28.89}{13.5} \approx 2.14$$

Therefore we have the linear function

$R(g) = 2.14g + 0 = 2.14g$.

If $g = 11.2$, then $R = (2.14)(11.2) = \$23.97$.

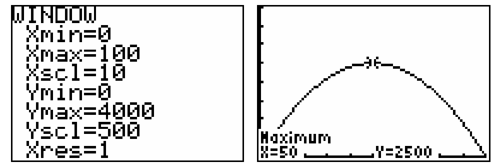
84. Since there are 200 feet of border, we know that $2x + 2y = 200$. The area is to be maximized, so $A = x \cdot y$. Solving the perimeter formula for y:

$2x + 2y = 200 \rightarrow 2y = 200 - 2x \rightarrow y = 100 - x$

The area function is:

$A(x) = x(100 - x) = -x^2 + 100x$

The maximum value occurs at the vertex:



The pond should be 50 feet by 50 feet for maximum area.

85. Let x represent the length and y represent the width of the rectangle.

$2x + 2y = 20 \quad \rightarrow \quad y = 10 - x$.

$x \cdot y = 16 \quad \rightarrow \quad x(10 - x) = 16$.

Solving the area equation:

$10x - x^2 = 16 \rightarrow x^2 - 10x + 16 = 0$

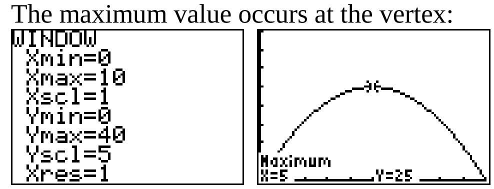
$(x - 8)(x - 2) = 0 \rightarrow x = 8$ or $x = 2$

The length and width of the rectangle are 8 feet by 2 feet.

86. The area function is:

$A(x) = x(10 - x) = -x^2 + 10x$

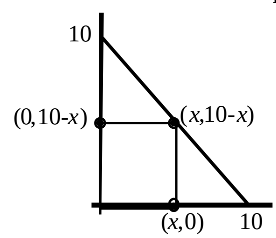
The maximum value occurs at the vertex:



The maximum area is:

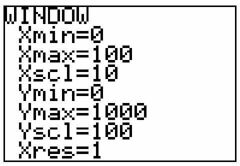
$A(5) = -(5)^2 + 10(5)$

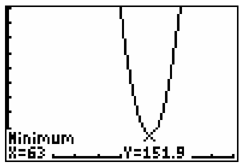
$= -25 + 50 = 25$ square units



87. $C(x) = 4.9x^2 - 617.40x + 19,600$;
$a = 4.9, b = -617.40, c = 19,600.$
Since $a = 4.9 > 0,$ the graph opens up, so the vertex is a minimum point.

a. The minimum marginal cost occurs at $x = 63$.





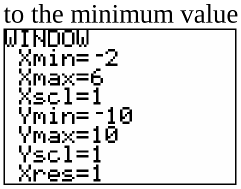
b. The minimum marginal cost is

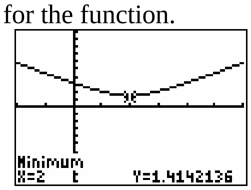
$$C\left(\frac{-b}{2a}\right) = C(63)$$
$$= 4.9(63)^2 - (617.40)(63) + 19600$$
$$= \$151.90$$

88. Let $P = (3,1)$ and $Q = (x, y) = (x, x)$.

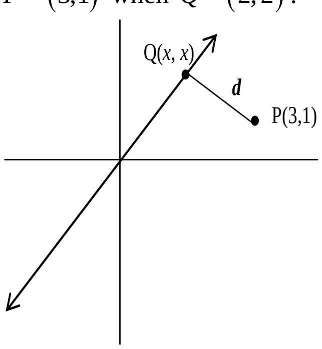
$$d(P,Q) = \sqrt{(x-3)^2 + (x-1)^2}$$
$$d^2(x) = (x-3)^2 + (x-1)^2$$
$$= x^2 - 6x + 9 + x^2 - 2x + 1$$
$$d^2(x) = 2x^2 - 8x + 10$$

Since $d^2(x) = 2x^2 - 8x + 10$ is a quadratic function with $a = 2 > 0$, the vertex corresponds to the minimum value for the function.





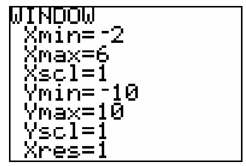
The vertex occurs at $x = 2$. Therefore the point Q on the line $y = x$ will be closest to the point $P = (3,1)$ when $Q = (2,2)$.

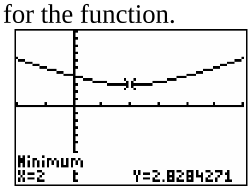


89. Let $P = (4,1)$ and $Q = (x, y) = (x, x+1)$.

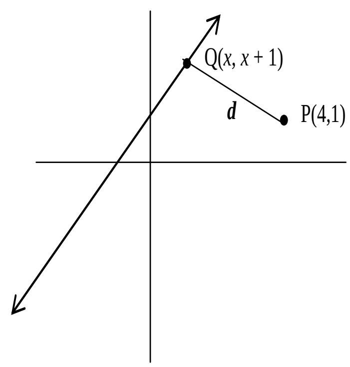
$$d(P,Q) = \sqrt{(x-4)^2 + (x+1-1)^2}$$
$$\rightarrow d^2(x) = (x-4)^2 + x^2$$
$$= x^2 - 8x + 16 + x^2$$
$$\therefore d^2(x) = 2x^2 - 8x + 16$$

Since $d^2(x) = 2x^2 - 8x + 16$ is a quadratic function with $a = 2 > 0$, the vertex corresponds to the minimum value for the function.

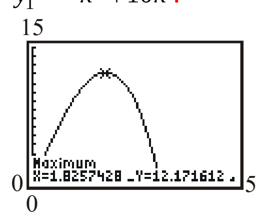




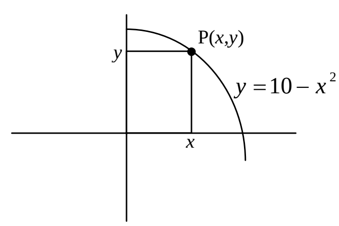
The vertex occurs at $x = 2$. Therefore the point Q on the line $y = x + 1$ will be closest to the point $P = (4,1)$ when $Q = (2,3)$.



90. The area function is:
$A(x) = x(10 - x^2) = -x^3 + 10x$
$y_1 = -x^3 + 10x$.



The maximum area is:
$A(1.83) = -(1.83)^3 + 10(1.83) \approx 12.17$ sq. units.



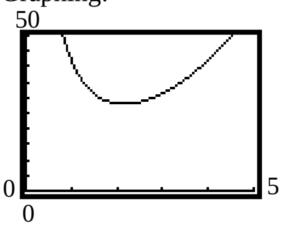
91. a. $x^2h = 10 \quad \Rightarrow \quad h = \dfrac{10}{x^2}$

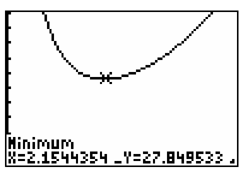
$$A(x) = 2x^2 + 4xh$$
$$= 2x^2 + 4x\left(\frac{10}{x^2}\right)$$
$$= 2x^2 + \frac{40}{x}$$

b. $A(1) = 2 \cdot 1^2 + \dfrac{40}{1} = 2 + 40 = 42 \text{ ft}^2$

c. $A(2) = 2 \cdot 2^2 + \dfrac{40}{2} = 8 + 20 = 28 \text{ ft}^2$

d. Graphing:





The area is smallest when $x \approx 2.15$ feet.

92. a. We are given that the volume is 500 centimeters, so we have

$$V = \pi r^2 h = 500 \Rightarrow h = \frac{500}{\pi r^2}$$

$$\text{Total Cost} = \text{cost}_{\text{top}} + \text{cost}_{\text{bottom}} + \text{cost}_{\text{body}}$$
$$= 2\left(\text{cost}_{\text{top}}\right) + \text{cost}_{\text{body}}$$
$$= 2\left(\text{area}_{\text{top}}\right)\left(\text{cost per area}_{\text{top}}\right)$$
$$+\left(\text{area}_{\text{body}}\right)\left(\text{cost per area}_{\text{body}}\right)$$
$$= 2\left(\pi r^2\right)(0.06) + (2\pi r h)(0.04)$$
$$= 0.12\pi r^2 + 0.08\pi r h$$
$$= 0.12\pi r^2 + 0.08\pi r\left(\frac{500}{\pi r^2}\right)$$
$$= 0.12\pi r^2 + \frac{40}{r}$$
$$C(r) = 0.12\pi r^2 + \frac{40}{r}$$

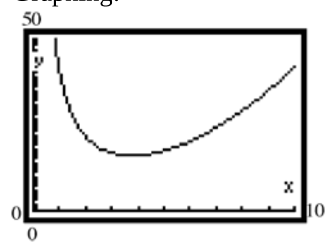
b. $C(4) = 0.12\pi(4)^2 + \dfrac{40}{4}$

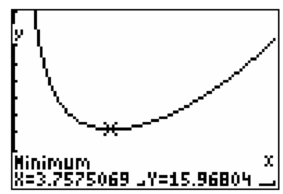
$$= 1.92\pi + 10 \approx \$16.03$$

c. $C(8) = 0.12\pi(8)^2 + \dfrac{40}{8}$

$$= 7.68\pi + 5 \approx \$29.13$$

d. Graphing:





The minimum cost occurs when $r \approx 3.76$ centimeters.

Chapter 2 Test

1. a. $\{(2,5),(4,6),(6,7),(8,8)\}$

This relation is a function because there are no ordered pairs that have the same first element and different second elements.

Domain: $\{2,4,6,8\}$

Range: $\{5,6,7,8\}$

b. $\{(1,3),(4,-2),(-3,5),(1,7)\}$

This relation is not a function because there are two ordered pairs that have the same first element but different second elements.

c. This relation is not a function because the graph fails the vertical line test.

d. This relation is a function because it passes the vertical line test.

Domain: $\mathbb{R}$ (all real numbers)

Range: $\{y \mid y \geq 2\}$

2. $f(x)=\sqrt{4-5x}$

The function tells us to take the square root of $4-5x$. Only nonnegative numbers have real square roots so we need $4-5x\ge 0$.

$$4-5x\ge 0$$
$$4-5x-4\ge 0-4$$
$$-5x\ge -4$$
$$\frac{-5x}{-5}\le\frac{-4}{-5}$$
$$x\le\frac{4}{5}$$

Domain: $\left\{x\mid x\le\frac{4}{5}\right\}$

$$f(-1)=\sqrt{4-5(-1)}=\sqrt{4+5}=\sqrt{9}=3$$

3. $g(x)=\dfrac{x+2}{|x+2|}$

The function tells us to divide $x+2$ by $|x+2|$. Division by 0 is undefined, so the denominator can never equal 0. This means that $x\ne -2$.

Domain: $\{x\mid x\ne -2\}$

$$g(-1)=\frac{(-1)+2}{|(-1)+2|}=\frac{1}{|1|}=1$$

4. $h(x)=\dfrac{x-4}{x^2+5x-36}$

The function tells us to divide $x-4$ by $x^2+5x-36$. Since division by 0 is not defined, we need to exclude any values which make the denominator 0.

$$x^2+5x-36=0$$
$$(x+9)(x-4)=0$$
$$x=-9 \text{ or } x=4$$

Domain: $\{x\mid x\ne -9, x\ne 4\}$

(note: there is a common factor of $x-4$ but we must determine the domain prior to simplifying)

$$h(-1)=\frac{(-1)-4}{(-1)^2+5(-1)-36}=\frac{-5}{-40}=\frac{1}{8}$$

5. a. To find the domain, note that all the points on the graph will have an x-coordinate between -5 and 5, inclusive. To find the range, note that all the points on the graph will have a y-coordinate between -3 and 3, inclusive.

Domain: $\{x\mid -5\le x\le 5\}$

Range: $\{y\mid -3\le y\le 3\}$

b. The intercepts are $(0,2)$, $(-2,0)$, and $(2,0)$.

x-intercepts: 2, -2

y-intercept: 2

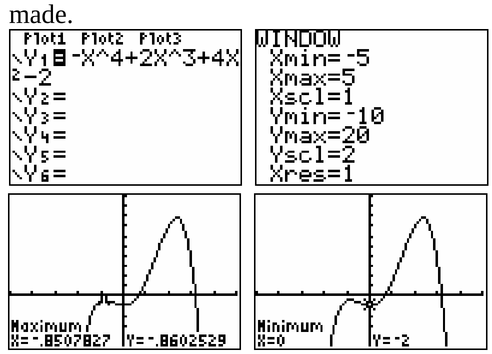
c. $f(1)$ is the value of the function when $x=1$. According to the graph, $f(1)=3$.

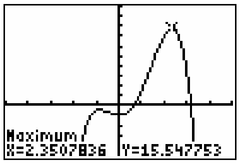
d. Since $(-5,-3)$ and $(3,-3)$ are the only points on the graph for which $y=f(x)=-3$, we have $f(x)=-3$ when $x=-5$ and $x=3$.

e. To solve $f(x)<0$, we want to find x-values such that the graph is below the x-axis. The graph is below the x-axis for values in the domain that are less than -2 and greater than 2. Therefore, the solution set is $\{x\mid -5\le x<-2 \text{ or } 2<x\le 5\}$. In interval notation we would write the solution set as $[-5,-2)\cup(2,5]$.

6. $f(x)=-x^4+2x^3+4x^2-2$

We set Xmin = -5 and Xmax = 5. The standard Ymin and Ymax will not be good enough to see the whole picture so some adjustment must be made.





We see that the graph has a local maximum of -0.86 (rounded to two places) when $x=-0.85$ and another local maximum of 15.55 when $x=2.35$. There is a local minimum of -2 when $x=0$. Thus, we have

Local maxima: $f(-0.85)\approx -0.86$

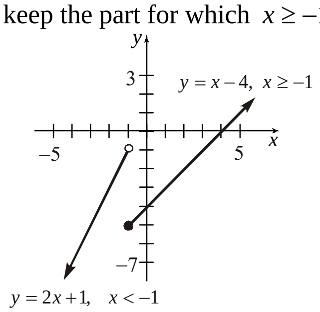
$f(2.35)\approx 15.55$

Local minima: $f(0)=-2$

The function is increasing on the intervals $(-5,-0.85)$ and $(0,2.35)$ and decreasing on the intervals $(-0.85,0)$ and $(2.35,5)$.

7. **a.** $f(x)=\begin{cases}2x+1 & x<-1\\ x-4 & x\ge -1\end{cases}$

To graph the function, we graph each "piece". First we graph the line $y=2x+1$ but only keep the part for which $x<-1$. Then we plot the line $y=x-4$ but only keep the part for which $x\ge -1$.



b. To find the intercepts, notice that the only piece that hits either axis is $y=x-4$.

$y=x-4$	$y=x-4$
$y=0-4$	$0=x-4$
$y=-4$	$4=x$

The intercepts are $(0,-4)$ and $(4,0)$.

c. To find $g(-5)$ we first note that $x=-5$ so we must use the first "piece" because $-5<-1$.

$g(-5)=2(-5)+1=-10+1=-9$

d. To find $g(2)$ we first note that $x=2$ so we must use the second "piece" because $2\ge -1$.

$g(2)=2-4=-2$

8. The average rate of change from 3 to x is given by

$$\frac{\Delta y}{\Delta x}=\frac{f(x)-f(3)}{x-3}\qquad x\ne 3$$
$$=\frac{(3x^2-2x+4)-(3(3)^2-2(3)+4)}{x-3}$$
$$=\frac{3x^2-2x+4-25}{x-3}$$
$$=\frac{3x^2-2x-21}{x-3}$$
$$=\frac{(x-3)(3x+7)}{x-3}$$
$$=3x+7\qquad x\ne 3$$

9. **a.** $f-g=(2x^2+1)-(3x-2)$

$=2x^2+1-3x+2$

$=2x^2-3x+3$

b. $f\cdot g=(2x^2+1)(3x-2)$

$=6x^3-4x^2+3x-2$

c. $f(x+h)-f(x)$

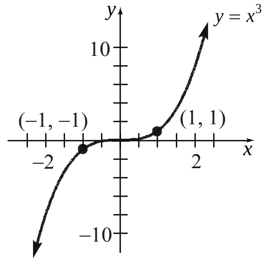
$=(2(x+h)^2+1)-(2x^2+1)$

$=(2(x^2+2xh+h^2)+1)-(2x^2+1)$

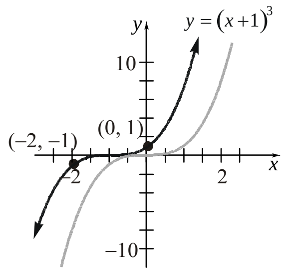
$=2x^2+4xh+2h^2+1-2x^2-1$

$=4xh+2h^2$

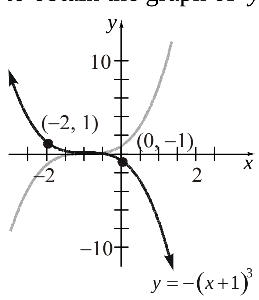
10. a. The basic function is $y = x^3$ so we start with the graph of this function.



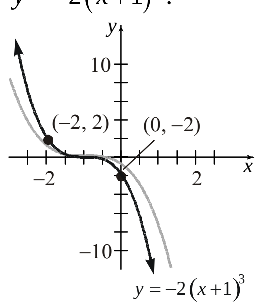
Next we shift this graph 1 unit to the left to obtain the graph of $y = (x+1)^3$.



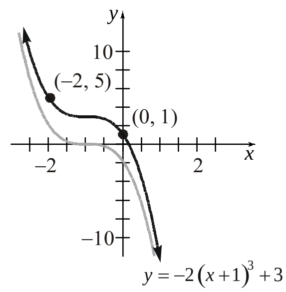
Next we reflect this graph about the x-axis to obtain the graph of $y = -(x+1)^3$.



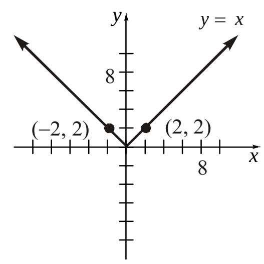
Next we stretch this graph vertically by a factor of 2 to obtain the graph of $y = -2(x+1)^3$.



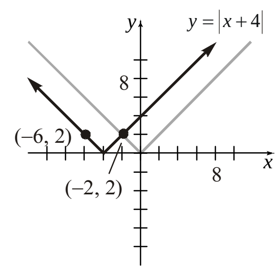
The last step is to shift this graph up 3 units to obtain the graph of $y = -2(x+1)^3 + 3$.



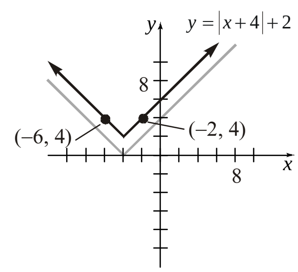
b. The basic function is $y = |x|$ so we start with the graph of this function.



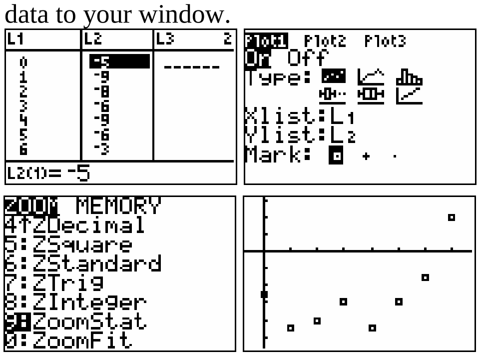
Next we shift this graph 4 units to the left to obtain the graph of $y = |x+4|$.



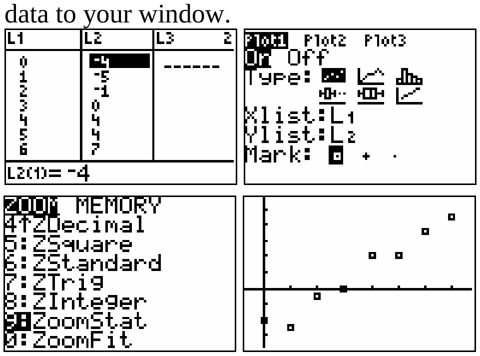
Next we shift this graph up 2 units to obtain the graph of $y = |x+4| + 2$.



11. a. Graph of Set A:
Enter the x-values in L1 and the y-values in L2. Make a scatter diagram using STATPLOT and press [ZOOM] [9] to fit the data to your window.

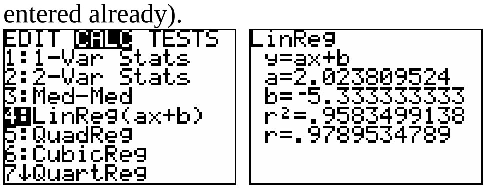


Graph of Set B:
Enter the x-values in L1 and the y-values in L2. Make a scatter diagram using STATPLOT and press [ZOOM] [9] to fit the data to your window.



From the graphs, it appears that Set B is more linear. Set A has too much curvature.

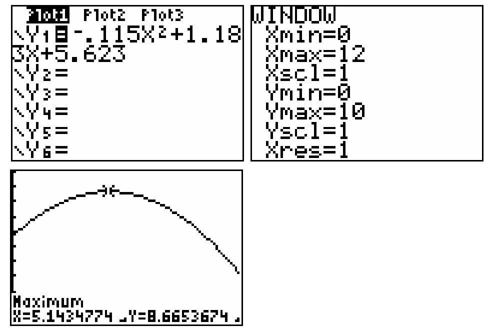
b. Set B appeared to be the most linear so we will use that data set.
Press [STAT] [▹] [4] [ENTER] to get the equation of the line (assuming the data is entered already).



The line of best fit for Set B is roughly $y = 2.02x - 5.33$.

12. a. $r(x) = -0.115x^2 + 1.183x + 5.623$

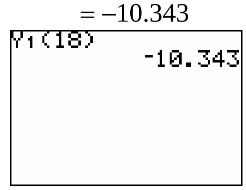
For the years 1992 to 2004, we have values of *x* between 0 and 12. Therefore, we can let Xmin = 0 and Xmax = 12. Since *r* is the interest rate as a percent, we can try letting Ymin = 0 and Ymax = 10.



The highest rate during this period appears to be 8.67%, occurring in 1997 ($x \approx 5$).

b. For 2010, we have $x = 2010 - 1992 = 18$.

$$r(18) = -0.115(18)^2 + 1.183(18) + 5.623$$
$$= -10.343$$



The model predicts that the interest rate will be -10.343%. This is not a reasonable value since it implies that the bank would be paying interest to the borrower.

13. a. Let x = width of the rink in feet. Then the length of the rectangular portion is given by $2x - 20$. The radius of the semicircular portions is half the width, or $r = \frac{x}{2}$.

To find the volume, we first find the area of the surface and multiply by the thickness of the ice. The two semicircles can be combined to form a complete circle, so the area is given by

$$A = l \cdot w + \pi r^2$$
$$= (2x - 20)(x) + \pi\left(\frac{x}{2}\right)^2$$
$$= 2x^2 - 20x + \frac{\pi x^2}{4}$$

We have expressed our measures in feet so we need to convert the thickness to feet as well.

$0.75 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{0.75}{12} \text{ ft} = \frac{1}{16} \text{ ft}$

Now we multiply this by the area to obtain the volume. That is,

$$V(x) = \frac{1}{16}\left(2x^2 - 20x + \frac{\pi x^2}{4}\right)$$

$$V(x) = \frac{x^2}{8} - \frac{5x}{4} + \frac{\pi x^2}{64}$$

b. If the rink is 90 feet wide, then we have $x = 90$.

$$V(90) = \frac{90^2}{8} - \frac{5(90)}{4} + \frac{\pi(90)^2}{64} \approx 1297.61$$

The volume of ice is roughly 1297.61 ft^3.

Chapter 2 Projects

Project 1

a. Plan A1: Total cost = $49.99 x 24 = $1199.76
Plan A2: Total cost = $59.99 x 24 = $1439.76
Plan B1: Total cost = $39.99 x 24 = $959.76
Plan B2: Total cost = $49.99 x 24 = $ 1199.76
Plan C1: Total cost = $45.00 x 24 = $1080.00
Plan C2: Total cost = $60.00 x 24 = $1440.00

b. All plans allow for 2500 night and weekend minutes free, so the only minutes that will be considered here are 600:

A1: $49.99
A2: $59.99
B1: $39.99 + 0.45(150) = $107.49
B2: $49.99
C1: $45.00 + 0.40(300) = $165.00
C2: $60.00

The best plan here is either plan A1 or B2 at $49.99.

The only plan that changes price from above when the night and weekend minutes increase to 3500 is B1. It only has 3000 free night and weekend minutes.

B1: $39.99 + 0.45(150) + 0.45(500) = $332.49

The best plan is still either A1 or B2.

c. For 425 minutes, all of them have a fixed price except C1: $45.00 + 0.40(125) = $95.
The best priced plan is B1 at $39.99.

For 750 minutes:
A1: $49.99 + 0.45(150) = $117.49
A2: $59.99
B1: $39.99 + 0.45(300) = $174.99
B2: $49.99 + 0.40(150) = $109.99
C1: $45.00 + 0.40(450) = $247.50
C2: $60.00 + 0.40(50) = $80.00

d. $$\begin{matrix}\text{Monthly} \\ \text{cost}\end{matrix} = \begin{matrix}\text{Base} \\ \text{Price}\end{matrix} + \left(\begin{matrix}\text{charge per} \\ \text{minute}\end{matrix}\right)\left(\begin{matrix}\text{\# of min. over} \\ \text{those included}\end{matrix}\right)$$

A1: $C(x) = 49.99 + 0.45(x - 600)$

$$C(x) = \begin{cases} 49.99 & 0 \le x \le 600 \\ 0.45x - 220.01 & x > 600 \end{cases}$$

A2: $C(x) = 59.99 + 0.35(x - 900)$

$$C(x) = \begin{cases} 59.99 & 0 \le x \le 900 \\ 0.35x - 255.01 & x > 900 \end{cases}$$

B1: $C(x) = 39.99 + 0.45(x - 450)$

$$C(x) = \begin{cases} 39.99 & 0 \le x \le 450 \\ 0.45x - 162.01 & x > 450 \end{cases}$$

B2: $C(x) = 49.99 + 0.40(x - 600)$

$$C(x) = \begin{cases} 49.99 & 0 \le x \le 600 \\ 0.40x - 190.01 & x > 600 \end{cases}$$

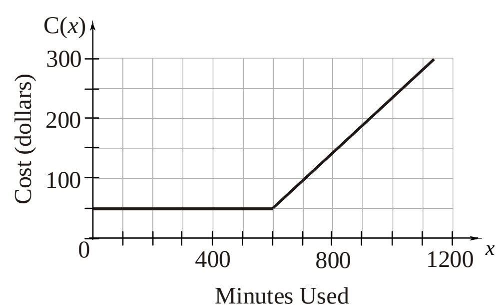
C1: $C(x) = 45.00 + 0.40(x - 300)$

$$C(x) = \begin{cases} 45.00 & 0 \le x \le 300 \\ 0.40x - 75 & x > 300 \end{cases}$$

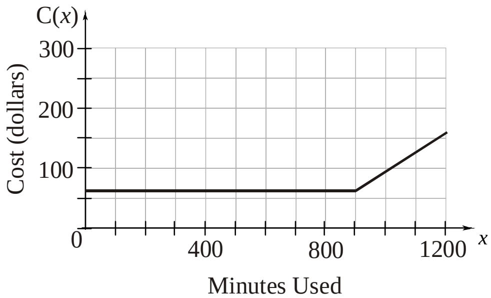
C2: $C(x) = 60.00 + 0.40(x - 700)$

$$C(x) = \begin{cases} 60.00 & 0 \le x \le 700 \\ 0.40x - 220 & x > 700 \end{cases}$$

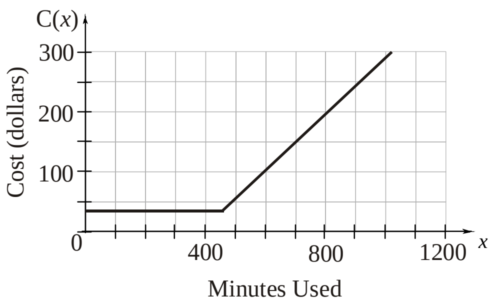
e. Graph for plan A1:



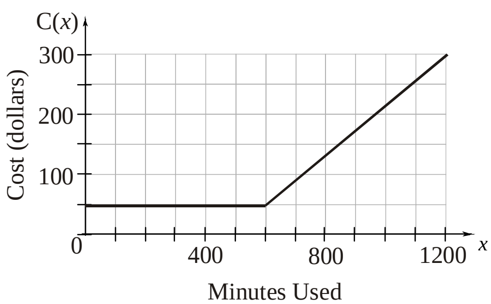
Graph for plan A2:



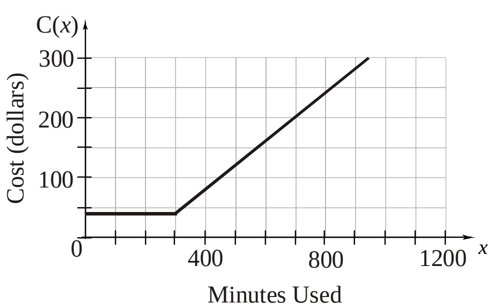
Graph for plan B1:



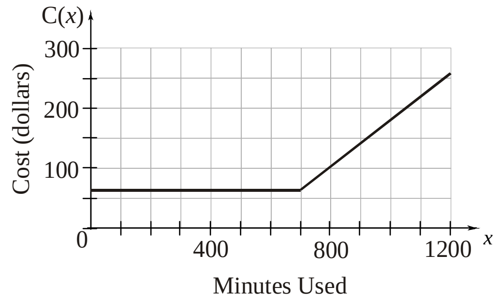
Graph for plan B2:



Graph for plan C1:



Graph for plan C2:



f. A1: $\dfrac{\$49.99}{600\,\text{min}} = \$0.083/\text{min}$

A2: $\dfrac{\$59.99}{900\,\text{min}} = \$0.067/\text{min}$

A2 is the better plan.

B1: $\dfrac{\$39.99}{450\,\text{min}} = \$0.089/\text{min}$

B2: $\dfrac{\$49.99}{600\,\text{min}} = \$0.083/\text{min}$

B2 is the better plan.

C1: $\dfrac{\$45.00}{300\,\text{min}} = \$0.15/\text{min}$

C2: $\dfrac{\$60.00}{700\,\text{min}} = \$0.086/\text{min}$

C2 is the better plan.

g. Out of A2, B2, and C2, the best plan to choose is A2 since its $/min rate is best.

h. Answers will vary.

<u>Project 2 (web)</u>

a. Silver: $C(x) = 20 + 0.16(x - 200) = 0.16x - 12$

$$C(x) = \begin{cases} 20 & 0 \le x \le 200 \\ 0.16x - 12 & x > 200 \end{cases}$$

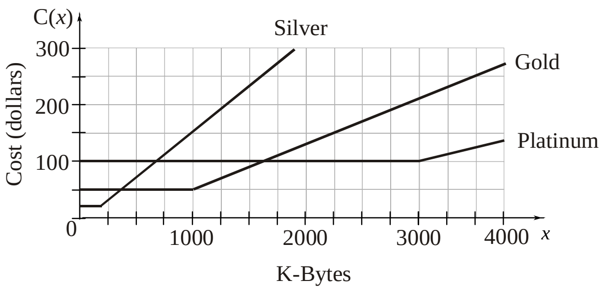
Gold: $C(x) = 50 + 0.08(x - 1000) = 0.08x - 30$

$$C(x) = \begin{cases} 50.00 & 0 \le x \le 1000 \\ 0.08x - 30 & x > 1000 \end{cases}$$

Platinum: $C(x) = 100 + 0.04(x - 3000)$

$= 0.04x - 20$

$$C(x) = \begin{cases} 100.00 & 0 \le x \le 3000 \\ 0.04x - 20 & x > 3000 \end{cases}$$



c. Let y = #K-bytes of service over the plan minimum.

Silver: $20+0.16y \le 50$

$0.16y \le 30$

$y \le 187.5$

Silver is the best up to $187.5+200=387.5$ K-bytes of service.

Gold: $50+0.08y \le 100$

$0.08y \le 50$

$y \le 625$

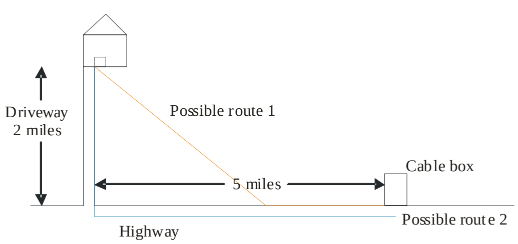
Gold is the best from 387.5 K-bytes to $625+1000=1625$ K-bytes of service.

Platinum: Platinum will be the best if more than 1625 K-bytes is needed.

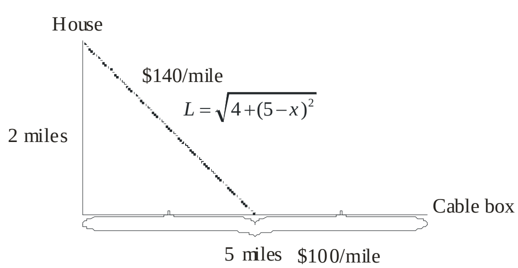
d. Answers will vary.

Project 3 (web)

a.



b.



$C(x)=100x+140L$

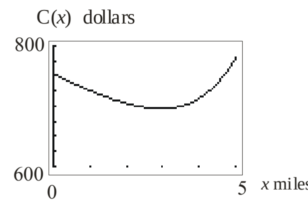
$C(x)=100x+140\sqrt{4+(5-x)^2}$

c.

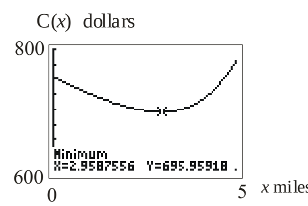
x	$C(x)$
0	$100(0)+140\sqrt{4+25} \approx \$753.92$
1	$100(1)+140\sqrt{4+16} \approx \$726.10$
2	$100(2)+140\sqrt{4+9} \approx \$704.78$
3	$100(3)+140\sqrt{4+4} \approx \$695.98$
4	$100(4)+140\sqrt{4+1} \approx \$713.05$
5	$100(5)+140\sqrt{4+0} = \$780.00$

The choice where the cable goes 3 miles down the road then cutting up to the house seems to yield the lowest cost.

d. Since all of the costs are less than $800, there would be a profit made with any of the plans.



Using the MINIMUM function on the grapher, the minimum occurs at about x = 2.96.



The minimum cost occurs when the cable runs for 2.96 mile along the road.

f. $C(4.5) = 100(4.5) + 140\sqrt{4+(5-4.5)^2}$

$\approx \$738.62$

The cost for the Steven's cable would be $738.62.

g. 5000(738.62) = $3,693,100 State legislated
5000(695.96) = $3,479,800 cheapest cost
It will cost the company $213,300 more.

Project 4 (web)

a. $A = \pi r^2$

b. $r = 2.2t$

c. $r = 2.2(2) = 4.4$ ft
$r = 2.2(2.5) = 5.5$ ft

d. $A = \pi(4.4)^2 = 60.82\ ft^2$
$A = \pi(5.5)^2 = 95.03\ ft^2$

e. $A = \pi(2.2t)^2 = 4.84\pi t^2$

f. $A = 4.84\pi(2)^2 = 60.82\ ft^2$
$A = 4.84\pi(2.5)^2 = 95.03\ ft^2$

g. $\dfrac{A(2.5) - A(2)}{2.5-2} = \dfrac{95.03 - 60.82}{0.5} = 68.42\ ft/hr$

h. $\dfrac{A(3.5) - A(3)}{3.5-3} = \dfrac{186.27 - 136.85}{0.5} = 98.84\ ft/hr$

i. The average rate of change is increasing.

j. 150 yds = 450 ft
$r = 2.2t$
$t = \dfrac{450}{2.2} = 204.5$ hours

k. 6 miles = 31680 ft
Therefore, we need a radius of 15,840 ft.
$t = \dfrac{15,840}{2.2} = 7200$ hours

Cumulative Review 1-2

1. $-5x + 4 = 0$
$-5x = -4$
$x = \dfrac{-4}{-5} = \dfrac{4}{5}$
The solution set is $\left\{\dfrac{4}{5}\right\}$.

2. $x^2 - 7x + 12 = 0$
$(x-4)(x-3) = 0 \Rightarrow x = 4, x = 3$
The solution set is $\{3, 4\}$.

3. $3x^2 - 5x - 2 = 0$
$(3x+1)(x-2) = 0 \Rightarrow x = -\dfrac{1}{3}, x = 2$
The solution set is $\left\{-\dfrac{1}{3}, 2\right\}$.

4. $4x^2 + 4x + 1 = 0$
$(2x+1)(2x+1) = 0 \Rightarrow x = -\dfrac{1}{2}$
The solution set is $\left\{-\dfrac{1}{2}\right\}$.

5. $4x^2 - 2x + 4 = 0 \Rightarrow 2x^2 - x + 2 = 0$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(2)}}{2(2)}$$
$$= \frac{1 \pm \sqrt{1-16}}{4} = \frac{1 \pm \sqrt{-15}}{4}$$
no real solution

6. $\sqrt[3]{1-x} = 2$
$\left(\sqrt[3]{1-x}\right)^3 = 2^3$
$1 - x = 8$
$-x = 7$
$x = -7$
The solution set is $\{-7\}$.

7. $\sqrt[5]{1-x} = 2$
$\left(\sqrt[5]{1-x}\right)^5 = 2^5$
$1 - x = 32$
$-x = 31$
$x = -31$
The solution set is $\{-31\}$.

8. $|2 - 3x| = 1$
$2 - 3x = 1$ or $2 - 3x = -1$
$-3x = -1$ or $-3x = -3$
$x = \dfrac{1}{3}$ or $x = 1$
The solution set is $\left\{\dfrac{1}{3}, 1\right\}$.

9. $4x^2 - 2x + 4 = 0 \Rightarrow 2x^2 - x + 2 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(2)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{1-16}}{4} = \frac{1 \pm \sqrt{-15}}{4} = \frac{1 \pm \sqrt{15}i}{4}$$

The solution set is $\left\{\frac{1-\sqrt{15}i}{4}, \frac{1+\sqrt{15}i}{4}\right\}$.

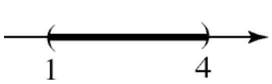
10. $-2 < 3x - 5 < 7$

$3 < 3x < 12$

$\frac{3}{3} < x < \frac{12}{3}$

$1 < x < 4$

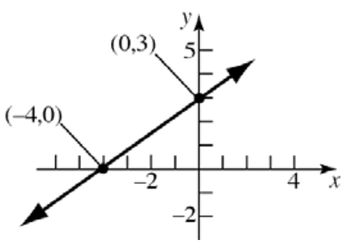
$\{x \mid 1 < x < 4\}$ or $(1, 4)$



11. $-3x + 4y = 12 \Rightarrow 4y = 3x + 12$

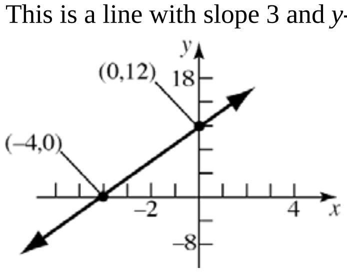
$y = \frac{3}{4}x + 3$

This is a line with slope $\frac{3}{4}$ and y-intercept (0, 3).



12. $y = 3x + 12$

This is a line with slope 3 and y-intercept (0,12).



13. $x^2 + y^2 + 2x - 4y + 4 = 0$

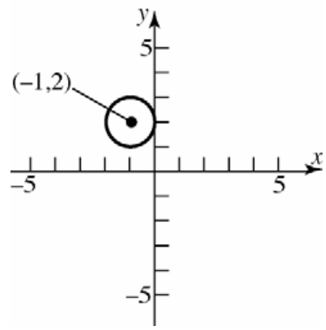
$x^2 + 2x + y^2 - 4y = -4$

$(x^2 + 2x + 1) + (y^2 - 4y + 4) = -4 + 1 + 4$

$(x+1)^2 + (y-2)^2 = 1$

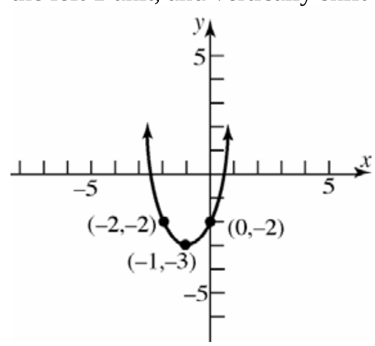
$(x+1)^2 + (y-2)^2 = 1^2$

This is a circle with center (–1,2) and radius 1.



14. $y = (x+1)^2 - 3$

Using the graph of $y = x^2$, horizontally shift to the left 1 unit, and vertically shift down 3 units.



15. **a.** Domain: $\{x \mid -4 \le x \le 4\}$

Range: $\{y \mid -1 \le y \le 3\}$

b. Intercepts: $(-1, 0)$, $(0, -1)$, $(1, 0)$

x-intercepts: $-1, 1$

y-intercept: -1

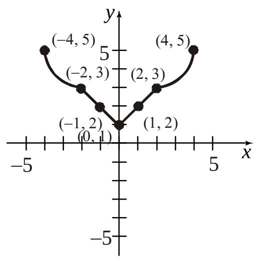
c. The graph is symmetric with respect to the y-axis.

d. When $x = 2$, the function takes on a value of 1. Therefore, $f(2) = 1$.

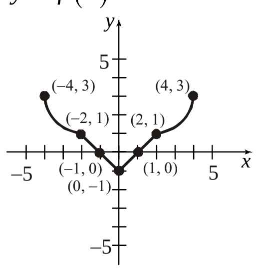
e. The function takes on the value 3 at $x = -4$ and $x = 4$.

f. $f(x) < 0$ means that the graph lies below the x-axis. This happens for *x* values between -1 and 1. Thus, the solution set is $\{x \mid -1 < x < 1\}$.

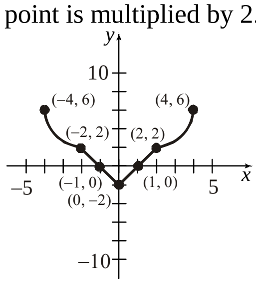
g. The graph of $y = f(x) + 2$ is the graph of $y = f(x)$ but shifted up 2 units.



h. The graph of $y = f(-x)$ is the graph of $y = f(x)$ but reflected about the y-axis.



i. The graph of $y = 2f(x)$ is the graph of $y = f(x)$ but stretched vertically by a factor of 2. That is, the coordinate of each point is multiplied by 2.



j. Since the graph is symmetric about the y-axis, the function is even.

k. The function is increasing on the open interval $(0, 4)$.

l. The function is decreasing on the open interval $(-4, 0)$.

m. There is a local minimum of -1 at $x = 0$. There are no local maxima.

n. $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{3 - 0}{3} = \dfrac{3}{3} = 1$

The average rate of change of the function from 1 to 4 is 1.

16.
$$\begin{aligned} d(P,Q) &= \sqrt{(-1-4)^2 + (3-(-2))^2} \\ &= \sqrt{(-5)^2 + (5)^2} \\ &= \sqrt{25+25} \\ &= \sqrt{50} = 5\sqrt{2} \end{aligned}$$

17. $y = x^3 - 3x + 1$

(a) $(-2, -1)$

$(-2)^3 - (3)(-2) + 1 = -8 + 6 + 1 = -1$

$(-2, -1)$ is on the graph.

(b) $(2, 3)$

$(2)^3 - (3)(2) + 1 = 8 - 6 + 1 = 3$

$(2, 3)$ is on the graph.

(c) $(3, 1)$

$(3)^3 - (3)(3) + 1 = 27 - 9 + 1 = 19 \neq 1$

$(3, 1)$ is not on the graph.

18. $y = 3x^2 + 14x - 5$

x-intercept(s): solve $3x^2 + 14x - 5 = 0$

$(3x - 1)(x + 5) = 0 \Rightarrow x = \dfrac{1}{3}, x = -5$

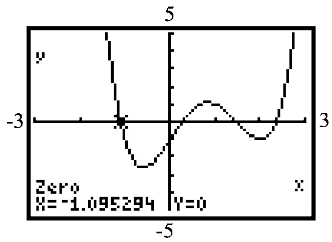
the x-intercepts are: $(-5, 0); \left(\dfrac{1}{3}, 0\right)$

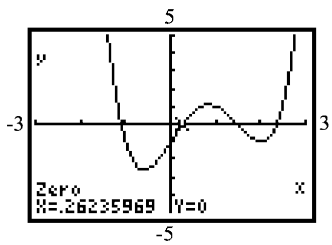
y-intercept: let $x = 0$

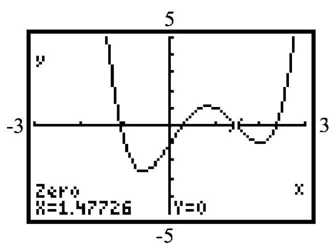
$y = 0^2 + 14(0) - 5 = -5$

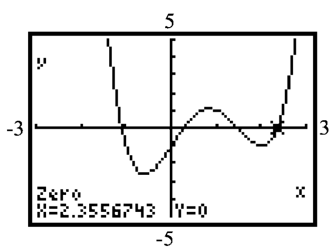
the y-intercept is: $(0, -5)$

19. Use ZERO (or ROOT) on the graph of $y_1 = x^4 - 3x^3 + 4x - 1$.









The solution set is $\{-1.10, 0.26, 1.48, 2.36\}$.

20. Perpendicular to $y = 2x + 1$;

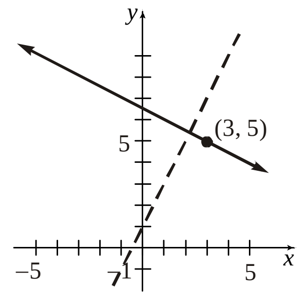
Slope of perpendicular = $-\frac{1}{2}$; Containing $(3,5)$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{1}{2}(x - 3)$$

$$y - 5 = -\frac{1}{2}x + \frac{3}{2}$$

$$y = -\frac{1}{2}x + \frac{13}{2}$$



21. Yes, each x corresponds to exactly 1 y.

22. $f(x) = x^2 - 4x + 1$

a. $f(2) = (2)^2 - 4(2) + 1 = 4 - 8 + 1 = -3$

b.
$$\begin{aligned} f(x) + f(2) &= x^2 - 4x + 1 + (2)^2 - 4(2) + 1 \\ &= x^2 - 4x + 1 + 4 - 8 + 1 \\ &= x^2 - 4x - 2 \end{aligned}$$

c. $f(-x) = (-x)^2 - 4(-x) + 1 = x^2 + 4x + 1$

d. $-f(x) = -\left(x^2 - 4x + 1\right) = -x^2 + 4x - 1$

e.
$$\begin{aligned} f(x+2) &= (x+2)^2 - 4(x+2) + 1 \\ &= x^2 + 4x + 4 - 4x - 8 + 1 \\ &= x^2 - 3 \end{aligned}$$

f. $\dfrac{f(x+h) - f(x)}{h}, h \neq 0$

$$\begin{aligned} &\frac{f(x+h) - f(x)}{h} \\ &= \frac{(x+h)^2 - 4(x+h) + 1 - \left(x^2 - 4x + 1\right)}{h} \\ &= \frac{x^2 + 2xh + h^2 - 4x - 4h + 1 - x^2 + 4x - 1}{h} \\ &= \frac{2xh + h^2 - 4h}{h} \\ &= \frac{h(2x + h - 4)}{h} = 2x + h - 4 \end{aligned}$$

23. $h(z) = \dfrac{3z-1}{z^2-6z-7}$

The denominator cannot be zero:

$$z^2 - 6z - 7 \neq 0$$
$$(z+1)(z-7) \neq 0$$
$$z \neq -1 \text{ or } 7$$

Domain: $\{z \mid z \neq -1,\ z \neq 7\}$

24. Yes, since the graph passes the Vertical Line Test.

25. $f(x) = \dfrac{x}{x+4}$

a. $f(1) = \dfrac{1}{1+4} = \dfrac{1}{5} \neq \dfrac{1}{4}$

$\left(1, \dfrac{1}{4}\right)$ is not on the graph of f

b. $f(-2) = \dfrac{-2}{-2+4} = \dfrac{-2}{2} = -1$

$(-2, -1)$ is on the graph of f

c. Solve for x:

$$\frac{x}{x+4} = 2$$
$$x = 2(x+4)$$
$$x = 2x + 8$$
$$-8 = x$$

$(-8, 2)$ is on the graph of f .