CHAPTER 1

END-OF-CHAPTER PROBLEMS

Analysis and engineering design

1.1 Essay

1.2 A 1-m long cantilevered beam of rectangular cross section carries a uniform load of $w = 15$ kN/m. The design specification calls for a 5-mm maximum deflection of the end of the beam. The beam is to be constructed of fir ($E = 13$ GPa). By analysis, determine at least five combinations of beam height h and beam width b that meet the specification. Use the equation

$$y_{\max} = \frac{wL^4}{8EI}$$

where,

$y_{\max}$ = deflection of end of beam (m)
w = uniform loading (N/m)
L = beam length (m)
E = modulus of elasticity of beam (N/m^2)
$I = bh^3/12$ = moment of inertia of beam cross section (m^4)

Note: 1 Pa = 1 N/m^2, 1 kN = 10^3 N and 1 GPa = 10^9 Pa.

What design conclusions can you draw about the influence of beam height and width on the maximum deflection? Is the deflection more sensitive to h or b? If the beam were constructed of a different material, how would the deflection change? See Figure P1.2 for an illustration of the beam.

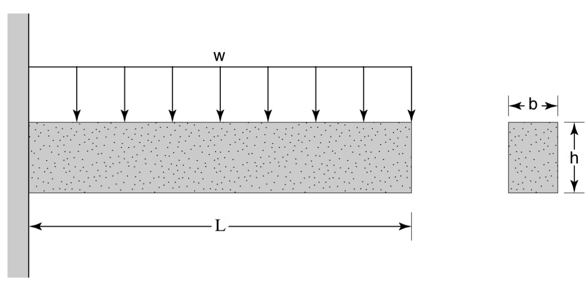

Figure P1.2

Solution

Given information:

$L = 1\text{m}$ $w = 15 \times 10^3$ N/m $y_{max} = 5$ mm = 0.005 m

$E = 13 \times 10^9$ Pa = 13×10^9 N/m^2

The deflection of the beam, y_{max}, will be calculated for one combination of h and b, and the rest can be done in the same manner.

By trial and error, we choose h = 200 mm and b = 100 mm. Thus, we have

$$y_{max} = \frac{wL^4}{8EI} = \frac{12\,wL^4}{8Ebh^3}$$

$$= \frac{12(15 \times 10^3 \text{ N/m})(1 \text{ m})^4}{8(13 \times 10^9 \text{ Pa})(0.1 \text{ m})(0.2 \text{ m})^3}$$

$$= 0.00216 \text{ m} = \underline{\underline{2.16 \text{ mm}}} < 5 \text{ mm}$$

An alternative way of doing the calculation is to set y_{max} equal to or below its maximum value, select an arbitrary value of h or b, and then find the other beam dimension.

The beam deflection is directly proportional to beam length, L, and inversely proportional to beam height, h, and beam width, b. Beam deflection is more sensitive to beam height, h, because this dimension is cubed. A beam can be made very stiff, i.e., the deflection can be made very small for a given load, by increasing the beam height. For example, if the beam height is doubled to 400 mm, the maximum deflection is only 0.270 mm.

If the beam were constructed of a different material, the modulus of elasticity, E, would be different. The beam deflection is inversely proportional to E. Modulus of elasticity is a mechanical property that denotes the stiffness of a material. A higher value of E means a higher stiffness. It is clear that a higher value of E results in a lower beam deflection. For example, if the beam were constructed of structural steel (E = 200 GPa) instead of fir, the beam deflection is only 0.141 mm.

Analysis and engineering failure

1.3 Essay

1.4 Essay