Chapter 0: Algebraic Concepts

Exercise 0.1

1. $12 \in \{1, 2, 3, 4, ...\}$

2. $5 \notin$ $\{x$: x is a natural number greater than $5\}$

3. $6 \notin \{1, 2, 3, 4, 5\}$

4. $3 \notin \varnothing$

5. $\{1, 2, 3, 4, 5, 6, 7\}$

6. $\{7, 8, 9\}$

7. $\{x$: x is a natural number greater than 2 and less than $8\}$

8. $\{x$: x is a natural number greater than $6\}$

9. $\varnothing \subseteq A$ since $\varnothing$ is a subset of every set. $A \subseteq B$ since every element of A is an element of B. $B \subseteq B$ since a set is always a subset of itself.

10. $\varnothing \subseteq A$ since $\varnothing$ is a subset of every set. $A \subseteq B$ since every element of A is an element of B. $B \subseteq B$ since a set is always a subset of itself.

11. No. $c \in A$ but $c \notin B$.

12. No. $12 \in A$ but $12 \notin B$.

13. $D \subseteq C$ since every element of D is an element of C.

14. $E \subseteq F$ since every element of E is an element of F.

15. $A \subseteq B$ and $B \subseteq A$. (Also $A = B$.)

16. $D \subseteq F$ and $F \subseteq D$. (Also $D = F$.)

17. Yes. $A \subseteq B$ and $B \subseteq A$. Thus, $A = B$.

18. $A \neq D$

19. No. $D \neq E$ because $4 \in E$ and $4 \notin D$.

20. $F = G$

21. A and B are disjoint since they have no elements in common. B and D are disjoint since they have no elements in common. C and D are disjoint.

22. $\varnothing$

23. $A \cap B = \{4, 6\}$ since 4 and 6 are elements of each set.

24. $A \cap B = \{a, d, e\}$, since a, d and e are elements of each set.

25. $A \cap B = \varnothing$ since they have no common elements.

26. $A \cap B = \{3\}$

27. $A \cup B = \{1, 2, 3, 4, 5\}$

28. $A \cup B = \{a, b, c, d, e, i, o, u\}$

29. $A \cup B = \{1, 2, 3, 4\}$ or $A \cup B = B$.

30. $A \cup B =$ $\{x$:x is a natural number not equal to $5\}$

For problems 31 - 42, we have $U = \{1, 2, 3, \ldots, 9, 10\}$.

31. $A' = \{4, 6, 7, 9, 10\}$ since these are the only elements in U that are not elements of A.

32. $B' = \{1, 2, 5, 6, 7, 9\}$
since these are the only elements in U that are not elements of B.

33. $B' = \{1, 2, 5, 6, 7, 9\}$
$A \cap B' = \{1, 2, 5, 7\}$

34. $A' = \{4, 6, 9, 10\}$
$B' = \{1, 2, 5, 6, 7, 9\}$
$A' \cap B' = \{6, 9\}$

35. $A \cup B = \{1, 2, 3, 4, 5, 7, 8, 10\}$
$(A \cup B)' = \{6, 9\}$

36. $A \cap B = \{3, 8\}$
$(A \cap B)' = \{1, 2, 4, 5, 6, 7, 9, 10\}$

37. $A' = \{4, 6, 9, 10\}$
$B' = \{1, 2, 5, 6, 7, 9\}$
$A' \cup B' = \{1, 2, 4, 5, 6, 7, 9, 10\}$

38. $A' = \{4, 6, 9, 10\}$
$B = \{3, 4, 8, 10\}$
$A' \cup B = \{3, 4, 6, 8, 9, 10\}$
$(A' \cup B)' = \{1, 2, 5, 7\}$

39. $B' = \{1, 2, 5, 6, 7, 9\}$,
$C' = \{1, 3, 5, 7, 9\}$
$A \cap B' = \{1, 2, 3, 5, 7, 8\} \cap \{1, 2, 5, 6, 7, 9\}$
$= \{1, 2, 5, 7\}$
$(A \cap B') \cup C' = \{1, 2, 3, 5, 7, 9\}$

40. $A = \{1, 3, 5, 8, 7, 2\}$
$B' = \{1, 2, 5, 6, 7, 9\}$
$C' = \{1, 3, 5, 7, 9\}$
$B' \cup C' = \{1, 2, 3, 5, 6, 7, 9\}$
$A \cap (B' \cup C') = \{1, 2, 3, 5, 7\}$

41. $B' = \{1, 2, 5, 6, 7, 9\}$
$A \cap B' = \{1, 2, 3, 5, 7, 8\} \cap \{1, 2, 5, 6, 7, 9\}$
$= \{1, 2, 5, 7\}$
$(A \cap B')' \cap C = \{3, 4, 6, 8, 9, 10\} \cap \{2, 4, 6, 8, 10\}$
$= \{4, 6, 8, 10\}$

42. $B \cup C = \{2, 3, 4, 6, 8, 10\}$
$A \cap (B \cup C) = \{2, 3, 8\}$

For problems 43 - 46, we have
$U = \{1, 2, 3, \ldots, 8, 9\}$.

43. $A - B = \{1, 3, 7, 9\} - \{3, 5, 8, 9\} = \{1, 7\}$

44. $A - B = \{1, 2, 3, 6, 9\} - \{1, 4, 5, 6, 7\} = \{2, 3, 9\}$

45. $A - B = \{2, 1, 5\} - \{1, 2, 3, 4, 5, 6\} = \varnothing$

46. $A - B = \{1, 2, 3, 4, 5\} - \{7, 8, 9\} = \{1, 2, 3, 4, 5\}$

47. a. $L = \{2000, 2001, 2004, 2005, 2006, 2007\}$ $H = \{2000, 2001, 2006, 2007, 2008\}$
$C = \{2001, 2002, 2003, 2008, 2009\}$
b. no
c. C' is the set of all years when the percentage change from low to high was 35% or less.
d. $H' = \{2002, 2003, 2004, 2005, 2009\}$
$C' = \{2000, 2004, 2005, 2006, 2007\}$
$H' \cup C' = \{2000, 2002, 2003, 2004, 2005, 2006, 2007, 2009\}$. $H' \cup C'$ is the set of years when the high was less than or equal to 11,000 or the percent change was less than or equal to 35%.
e. $L' = \{2002, 2003, 2008, 2009\}$
$L' \cap C = \{2002, 2003, 2008, 2009\}$.
$L' \cap C$ is the set of years when the low was less than or equal to 8,000 and the percent change was more than 35%.

48. a. $A = \{\text{O, L, P}\}$
$B = \{\text{L, P}\}$
$C = \{\text{O, M, P}\}$
b. $B \subseteq A$
c. $A \cap C = \{\text{O, P}\}$; this is the set of cities with at least 2,000,000 jobs in 2000 or 2025 and projected annual growth rates of at least 2.5%.
d. B' is the set of cities with less than 1,500,000 jobs in 2000.

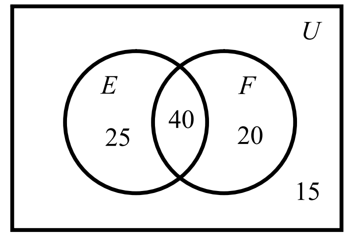
49. a. From the table, there are 100 white Republicans and 30 non-white Republicans who favor national health care, for a total of 130.
b. From the table, there are 350 + 40 Republicans, and 250 + 200 Democrats who favor national health care, for a total of 840.
c. From the table, there are 350 white Republicans, and 150 white Democrats and 20 non-whites who oppose national health care, for a total of 520.

50. a. From the table, 250 white Republicans and 150 white Democrats oppose national health care, for a total of 400.

b. From the table, there are 750 whites and there are 20 non-whites who oppose national health care. The total of this group is 770.

c. From the table, there are 200 non-white Democrats who favor national health care.

51. a. The key to solving this problem is to work from "the inside out". There are 40 aides in $E \cap F$. This leaves 65 – 40 = 25 aides who speak English but do not speak French. Also we have 60 – 40 = 20 aides who speak French but do not speak English. Thus there are 40 + 25 + 20 = 85 aides who speak English or French. This means there are 15 aides who do not speak English or French.



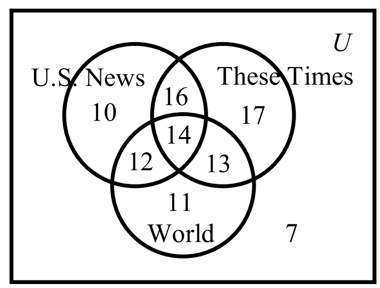
b. From the Venn diagram $E \cap F$ has 40 aides.

c. From the Venn diagram $E \cup F$ has 85 aides.

d. From the Venn diagram $E \cap F'$ has 25 aides.

52. There are 14 advertisers in the intersection of the sets. Since 30 advertised in *These Times* and *U.S. News* and we already have 14 in the center, 16 advertised in *These Times* and *U.S. News* and not in *World*. Since 26 advertised in *World* and *U.S. News* and we already have 14 in the center, 12 advertised in *World* and *U.S. News* and not in *These Times*. Since 27 advertised in *World* and *These Times* and we already have 14 in the middle, 13 advertised in *World* and *These Times* and not in *U.S. News*. 60 advertised in *These Times* and we have already accounted for 43, so 17 advertised in *These Times* only. 52 advertised in *U.S. News* and we have already accounted for 42, so 10 advertised in *U.S. News* only. 50 advertised in *World* and we have already accounted for 39, so 11 advertised in *World* only.

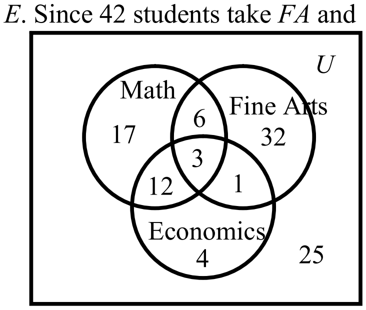
a. In the union of the 3 publications we have 10 + 16 + 17 + 14 + 12 + 13 + 11 = 93 advertisers. Thus, there are 100 – 93 = 7 who advertised in none of these publications.



b. There are 17 advertisers in the *These Times* circle that are not in an intersection.

c. In the union of *U.S. News* and *These Times* we have 10 + 12 + 16 + 14 + 17 + 13 = 82 advertisers.

53. Since 12 students take M and E but not FA, and 15 take M and E, 3 take all three classes. Since 9 students take M and FA and we have already counted 3, there are 6 taking M and FA which are not taking E. Since 4 students take E and FA and we have already counted 3, there is only 1 taking E and FA but not taking M also. Since 20 students take E and we already have 16 enrolled in E, this leaves 4 taking only E. Since 42 students take FA and we already have 10 enrolled in FA, this leaves 32 taking only FA. Since 38 students take M and we already have 21 enrolled in M, this leaves 17 taking only M.

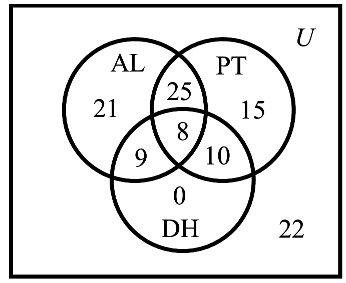


a. In the union of the 3 courses we have 17 + 12 + 3 + 6 + 32 + 1 + 4 = 75 students enrolled. Thus, there are 100 – 75 = 25 students who are not enrolled in any of these courses.

b. In $M \cup E$ we have 17 + 12 + 3 + 6 + 1 + 4 = 43 enrolled.

c. We have 17 + 32 + 4 = 53 students enrolled in exactly one of the courses.

54. Start by filling in the parts of the diagram for AL, since we have more information about it. 21 liked AL only. Since 30 liked AL but not PT, 9 liked AL or PT exclusively. 25 liked PT or AL but not DH, and 63 liked AL. That leaves 63 – (21 + 25 + 9) = 8 in the intersection of all 3. Since 18 liked PT and DH, only 10 liked PT and DH but not AL. Since 27 liked DH, 27 – (9 + 8 + 10) = 0 liked DH. And since 58 liked PT, 58 – (25 + 8 + 10) = 15 liked PT only.

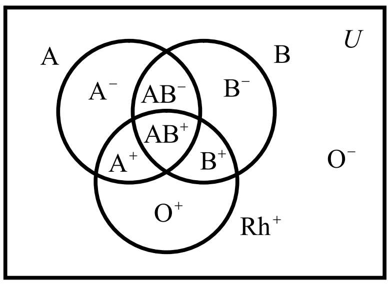


a. The number of students that liked PT or DH is 25 + 15 + 9 + 8 + 10 + 0 = 67.

b. The number that liked all three is 8.

c. The number that liked only DH is 0.

55. (**a**) and (**b**)



c. A^+ : 34%; B^+ : 9%; O^+ : 38%; AB^+ : 3%; O^- : 7%; A^- : 6%; B^- : 2%; AB^- : 1%

Exercise 0.2

1. a. Note that $-\dfrac{\pi}{10} = \pi \cdot \left(-\dfrac{1}{10}\right)$, where π is irrational and $-\dfrac{1}{10}$ is rational. The product of a rational number and an irrational number is an irrational number.

b. –9 is rational and an integer.

c. $\dfrac{9}{3} = \dfrac{3}{1} = 3$. This is a natural number, an integer, and a rational number.

d. Division by zero is meaningless.

2. a. $\dfrac{0}{6} = 0$ is rational and an integer.

b. rational

c. rational

d. rational

3. a. Commutative

b. Distributive

4. a. Associative

b. Additive identity

5. a. Multiplicative identity

b. Additive inverse

6. a. Multiplicative inverse

b. Commutative

7. $-6 < 0$

8. $2 > -20$

9. $-14 < -3$

10. $\pi > 3.14$

11. $0.333 < \dfrac{1}{3}\left(\dfrac{1}{3} = 0.3333\cdots\right)$

12. $\dfrac{1}{3} + \dfrac{1}{2} = \dfrac{5}{6}$

13. $|-3| + |5| > |-3+5|$

14. $|-9-3| = |-9| + |3|$ $(12 = 12)$

15. $-3^2 + 10 \cdot 2 = -3^2 + 20 = -9 + 20 = 11$

16. $(-3)^2 + 10 \cdot 2 = 9 + 20 = 29$

17. $\frac{4+2^2}{2}=\frac{4+4}{2}=\frac{8}{2}=4$

18. $\frac{(4+2)^2}{2}=\frac{6^2}{2}=\frac{36}{2}=18$

19. $\frac{16-(-4)}{8-(-2)}=\frac{16+4}{8+2}=\frac{20}{10}=2$

20. $\frac{(-5)(-3)-(-2)(3)}{-9+2}=\frac{15-(-6)}{-7}=\frac{15+6}{-7}$

$=\frac{21}{-7}=-3$

21. $\frac{|5-2|-|-7|}{|5-2|}=\frac{|3|-|-7|}{|3|}=\frac{3-7}{3}=-\frac{4}{3}$

22. $\frac{|3-|4-11||}{-|5^2-3^2|}=\frac{|3-|-7||}{-|25-9|}$

$=\frac{|3-7|}{-|16|}$

$=\frac{|-4|}{-16}$

$=\frac{4}{-16}=-\frac{1}{4}$

23. $\frac{(-3)^2-2\cdot 3+6}{4-2^2+3}=\frac{9-6+6}{4-4+3}=\frac{9}{3}=3$

24. $\frac{6^2-4(-3)(-2)}{6-6^2\div 4}=\frac{36-(-12)(-2)}{6-36\div 4}$

$=\frac{36-24}{6-9}$

$=\frac{12}{-3}$

$=-4$

25. $\frac{-4^2+5-2\cdot 3}{5-4^2}=\frac{-16+5-6}{5-16}=\frac{-17}{-11}=\frac{17}{11}$

26. $\frac{3-2(5-2)}{(-2)^2-2^2+3}=\frac{3-2\cdot 3}{4-4+3}=\frac{-3}{3}=-1$

27. The entire line

28. The interval notation corresponding to $x\ge 0$ is $[0,\infty)$.

29. (1, 3]; half-open interval

30. [–4, 3]; closed interval

31. (2, 10); open interval

32. $[2,\infty)$; half-open interval

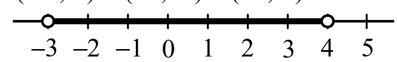
33. $-3\le x<5$

34. $x>-2$

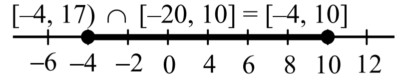
35. $x>4$

36. $0\le x<5$

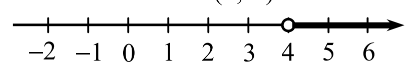
37. $(-\infty, 4)\cap(-3, \infty)=(-3, 4)$



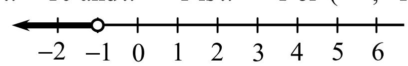
38. $[-4, 17)\cap[-20, 10]=[-4, 10]$



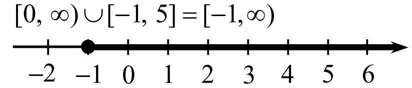
39. $x>4$ and $x\ge 0$ = $(4,\infty)$



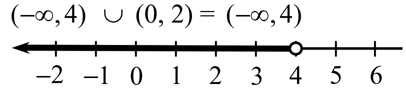
40. $x<10$ and $x<-1$ is $x<-1$ or $(-\infty,-1)$.



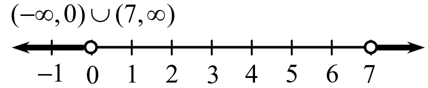
41. $[0, \infty)\cup[-1, 5]=[-1,\infty)$



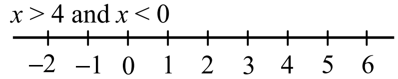
42. $(-\infty,4)\cup(0, 2)=(-\infty,4)$



43. $(-\infty,0)\cup(7,\infty)$



44. $x>4$ and $x<0$



The intersection is the empty set

45. –0.000038585

46. 0.404787025

47. 9122.387471

48. 11.80591621

49. $\dfrac{2500}{[(1.1^6)-1]}=\dfrac{2500}{0.771561}=3240.184509$

50. 1591.712652

51. a. $\$300.00+\$788.91=\$1088.91$

b. $0.25\left[1088.91-0.05(1088.91)\right]=\$258.62$

Retirement: $0.05(1088.91)=\$54.45$

Sales tax = Retirement = \$54.45

Local tax = $0.01(1088.91)=\$10.89$

Federal tax = $0.25(1088.91-54.45)=\$258.62$

Soc. Sec. tax = $0.0765(1088.91)=\$83.30$

Total Withholding = \$461.71

Take-home = $1088.91-461.71=\$627.20$

52. a. $t=2010-2000=10$

b. $E=5.03(10)^2+100(10)+1380$

$=\$2883$ billion

c. $t=2015-2000=15$

$E=50.3(15)^2+100(15)+1380$

$=\$4011.75$ billion

53. a. Formula (2) is a closer approximation.

$P=0.3179(6)+13.85=15.7574\%$

$P=0.0194(6)^3-0.1952(6)^2$

$+0.8282(6)+13.63$

$=15.7624\%$

b. (1): 17.665%; (2): 28.983%
Formula (2) seems too high, formula (1) seems more accurate.

54. a. $H=2.31(10.5)+31.26=55.515$ inches

Upper: $1.05(55.515)=58.29$ inches

Lower: $0.95(55.515)=52.74$ inches

$52.74\le H\le 58.29$

b. $H=2.31(5.75)+31.26=44.5425$ inches

Upper: $1.05(44.5425)=46.77$ inches

Lower: $0.95(44.5425)=42.32$ inches

$42.32\le H\le 46.77$

55. a. $\$82,401\le I\le 171,850$;

$\$171,851\le I\le \$373,650$;

$I>\$373,650$

b. $T=\$4681.25$ for $I=\$34,000$

$T=\$16,781.25$ for $I=\$82,400$

c. $[4681.25,\ 16,781.25]$

Exercise 0.3

1. $(-4)^4=(-4)(-4)(-4)(-4)=256$

2. $-5^3=-1\cdot5\cdot5\cdot5=-125$

3. $-2^6=-1\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2=-64$

4. $(-2)^5=(-2)(-2)(-2)(-2)(-2)=-32$

5. $3^{-2}=\dfrac{1}{3^2}=\dfrac{1}{9}$

6. $6^{-1}=\dfrac{1}{6}$

7. $-\left(\dfrac{3}{2}\right)^2=(-1)\left(\dfrac{3}{2}\right)\left(\dfrac{3}{2}\right)=-\dfrac{9}{4}$

8. $\left(\dfrac{2}{3}\right)^3=\dfrac{2^3}{3^3}=\dfrac{8}{27}$

9. $6^5\cdot6^3=6^{5+3}=6^8$

10. $8^4\cdot8^2\cdot8=8^{4+2+1}=8^7$

11. $\dfrac{10^8}{10^9}=10^{8-9}=10^{-1}=\dfrac{1}{10}$

12. $\dfrac{7^8}{7^3}=7^{8-3}=7^5$

13. $\dfrac{9^4\cdot9^{-7}}{9^{-3}}=\dfrac{9^{4+(-7)}}{9^{-3}}=\dfrac{9^{-3}}{9^{-3}}=9^{-3-(-3)}=9^0=1$

14. $\frac{5^4}{\left(5^{-2}\cdot 5^3\right)}=\frac{5^4}{5^{-2+3}}=\frac{5^4}{5^1}=5^{4-1}=5^3$

15. $\left(3^3\right)^3=3^{3\cdot 3}=3^9$

16. $\left(2^{-3}\right)^{-2}=2^{(-3)\cdot(-2)}=2^6$

17. $\left(\frac{2}{3}\right)^{-2}=\left(\frac{3}{2}\right)^2=\frac{9}{4}$

18. $\left(\frac{-2}{5}\right)^{-4}=\left(\frac{5}{-2}\right)^4=\left(-\frac{5}{2}\right)^4$

19. $(x^2)^{-3}=x^{2(-3)}=x^{-6}=\frac{1}{x^6}$

20. $x^{-4}=\frac{1}{x^4}$

21. $xy^{-2}z^0=x\cdot\frac{1}{y^2}\cdot 1=\frac{x}{y^2}$

22. $(xy^{-2})^0=1$

23. $x^3\cdot x^4=x^{3+4}=x^7$

24. $a^5\cdot a=a^{5+1}=a^6$

25. $x^{-5}\cdot x^3=x^{-5+3}=x^{-2}=\frac{1}{x^2}$

26. $y^{-5}\cdot y^{-2}=y^{-5+(-2)}=y^{-7}=\frac{1}{y^7}$

27. $\frac{x^8}{x^4}=x^{8-4}=x^4$

28. $\frac{a^5}{a^{-1}}=a^{5-(-1)}=a^{5+1}=a^6$

29. $\frac{y^5}{y^{-7}}=y^{5-(-7)}=y^{12}$

30. $\frac{y^{-3}}{y^{-4}}=y^{-3-(-4)}=y^{-3+4}=y^1=y$

31. $\left(x^4\right)^3=x^{3\cdot 4}=x^{12}$

32. $(y^3)^{-2}=y^{3(-2)}=y^{-6}=\frac{1}{y^6}$

33. $(xy)^2=x^2y^2$

34. $(2m)^3=2^3m^3=8m^3$

35. $\left(\frac{2}{x^5}\right)^4=\frac{2^4}{\left(x^5\right)^4}=\frac{16}{x^{5\cdot 4}}=\frac{16}{x^{20}}$

36. $\left(\frac{8}{a^3}\right)^3=\frac{8^3}{(a^3)^3}=\frac{512}{a^{3\cdot 3}}=\frac{512}{a^9}$

37. $\left(2x^{-2}y\right)^{-4}=2^{-4}x^8y^{-4}=\frac{x^8}{16y^4}$

38.
$$\begin{aligned}\left(-32x^5\right)^{-3}&=(-32)^{-3}\left(x^5\right)^{-3}\\&=\frac{1}{(-32)^3}x^{5(-3)}\\&=\frac{1}{-32768}\cdot x^{-15}\\&=-\frac{1}{32768x^{15}}\end{aligned}$$

39.
$$\begin{aligned}\left(-8a^{-3}b^2\right)\left(2a^5b^{-4}\right)&=-16a^{-3+5}b^{2-4}\\&=-16a^2b^{-2}\\&=-\frac{16a^2}{b^2}\end{aligned}$$

40.
$$\begin{aligned}\left(-3m^2y^{-1}\right)\left(2m^{-3}y^{-1}\right)&=-6m^{2+(-3)}y^{-1+(-1)}\\&=-6m^{-1}y^{-2}\\&=-6\left(\frac{1}{m}\right)\left(\frac{1}{y^2}\right)\\&=\frac{-6}{my^2}\end{aligned}$$