Set 1.2a

1

First 4 weeks: 2 weekend-roundtrips FYV-DEN-FYV and 2 weekend-roundtrips DEN-FYV-DEN. Week 5: 1 roundtrip.

2

Given a string of length L:

(1) h = .3L, w = .2L, Area = $.06L^2$
(2) h = .1L, w = .4L, Area = $.04L^2$

Solution (2) is better because the area is larger

3

$L = 2(w + h)$
$w = L/2 - h$

$z = wh = h(L/2 - h) = Lh/2 - h^2$

$\delta z/\delta h = L/2 - 2h = 0$

Thus, $h = L/4$ and $w = L/4$.

Solution is optimal because z is a concave function

4

(a) Let T = Total tie to move all four individuals to the other side of the river. the objective is to determine the transfer schedule that minimizes T.
(b) Let t = crossing time from one side to the other. Use codes 1, 2, 5, and 10 to represent Amy, Jim, John, and Kelly.

4 cont.

East	Crossing	West
5,10	(1,2)→ **(t = 2)**	1,2
1,5,10	**(t = 1)**←(1)	2
1	(5,10)→ **(t = 10)**	2,5,10
1,2	**(t = 2)**←(2)	5,10
none	(1,2)→ **(t = 2)**	1,2,5,10
Total = 2 + 1 + 10 + 2 + 2 = 17 minutes		

5

		Jim: Curve	Jim: Fast
Joe	Curve	.500	.200
	Fast	.100	.300

(a) Alternatives:
Jim: Throw curve of fast ball.
Joe: Prepare for curve or fast ball.

(b) Joe tries to improve his batting score and Jim tries to counter Joe's action by selecting a less favorable strategy. This means that neither player will be satisfied with a single (pure) strategy.

The problem is not an optimization situation in the familiar sense in which the objective is maximized or minimized. Instead, the conflicting situation requires a compromise solution in which neither layer is tempted to change strategy. Game theory (Chapter 14) provides such a solution.

Let L=ops. 1 and 2=20 sec, C=ops. 3 and 4=25 sec, U=op. 5=20 sec

Gant chart: L1+load horse 1, L2=load horse 2, etc.

one joist: 0---L1---**20**---C1---45----U1+L1---**85**----U2+L2----125---U1+L1---
165---U2+L2---*205*
20-L2-40 45---C2----70 **85**---C1---110 125---C2---140
165-C1-190
205---C2---230---U2---250

Total = 250
Loaders utilization=[250-(5+25)]/250=88%
Cutter utilization=[250-(20+15+15+15+15)]/250=68%

two joists: 0---2L1---**40**-----2C1-----**90**----2(U1+L1)---**170**----2C1----220---2U1-
--260
40---2L2---80 **90**---2C2----140 **170**---2U2---210

Total =260
Loaders utilization=[260-(10+10)]/260=92%
Cutter utilization=[260-(40+30+40)]/250=58%

three joists: 0---3L1---**60**------3C1------**135**------3C2---------210----3U2----270
60---3L2---120 **135**-----3U1-----195

Total =270
Loaders utilization=[270-(15+15)]/270=89%
Cutter utilization=[270-(60+60)]/270=56%

Recommendation: One joist at a time gives the smallest time. The problem has other alternatives that combine 1, 2, and 3 joists. Cutter utilization indicates that cutter represents the bottleneck.

7

10
8 9
5 6 7
1 2 3 4

(a) Alternative 1: Move dots 5, 6, and 7 below bottom row, move dots 8 and 9 below new 5, 6, and 7. Move 10 to the bottom. Number of moves = 6. Alternative 2: See part (b)

(b) Three moves: Move dot 1 up to the left of dot 8, dot 4 to the right of dot 9, and dot 10 below dots 2 and 3.

8

(a) Alternative 1: Break one end link of each chain and connect to another chain. Four breaks and re-solders, cost = 4 × (2 + 3) =20 cents. Alternative 2: See Part (b)

(b) Break three links in one chain and use them to connect the remaining three chains: Three breaks and re-solder, cost = 3 × (2 + 3) = 15 cents.

9

Represent the selected 2-digit number as $10x+y$. The corresponding square number is $10x+y-(x+y)=9x$. This means that the selected square will always be 9, 18, 27, …, or 81. By assigning zero dollars to these squares, the reward is always zero regardless of the rewards assigned to the remaining squares or the number of times the game is repeated..

CHAPTER 2

Modeling with Linear Programming

Set 2.1a

1

(a) $x_2 - x_1 \geq 1$ or $-x_1 + x_2 \geq 1$

(b) $x_1 + 2x_2 \geq 3$ and $x_1 + 2x_2 \leq 6$

(c) $x_2 \geq x_1$ or $x_1 - x_2 \leq 0$

(d) $x_1 + x_2 \geq 3$

(e) $\frac{x_2}{x_1 + x_2} \leq .5$ or $.5x_1 - .5x_2 \geq 0$

2

(a) $(x_1, x_2) = (1, 4)$

$(x_1, x_2) \geq 0$

$6 \times 1 + 4 \times 4 = 22 < 24$

$1 \times 1 + 2 \times 4 = 9 \nleq 6$ infeasible

(b) $(x_1, x_2) = (2, 2)$

$(x_1, x_2) \geq 0$

$6 \times 2 + 4 \times 2 = 20 < 24$

$1 \times 2 + 2 \times 2 = 6 = 6$

$-1 \times 2 + 1 \times 2 = 0 < 1$

$1 \times 2 = 2 = 2$

} feasible

$Z = 5 \times 2 + 4 \times 2 = \$18$

(c) $(x_1, x_2) = (3, 1.5)$

$x_1, x_2 \geq 0$

$6 \times 3 + 4 \times 1.5 = 24 = 24$

$1 \times 3 + 2 \times 1.5 = 6 = 6$

$-1 \times 3 + 1 \times 1.5 = -1.5 < 1$

$1 \times 1.5 = 1.5 < 2$

} feasible

$Z = 5 \times 3 + 4 \times 1.5 = \$21$

(d) $(x_1, x_2) = (2, 1)$

$x_1, x_2 \geq 0$

$6 \times 2 + 4 \times 1 = 16 < 24$

$1 \times 2 + 2 \times 1 = 4 < 6$

$-1 \times 2 + 1 \times 1 = -1 < 1$

$1 \times 1 = 1 < 2$

} feasible

$Z = 5 \times 2 + 4 \times 1 = \$14$

(e) $(x_1, x_2) = (2, -1)$

$x_1 \geq 0$, $x_2 < 0$, infeasible

Conclusion: (c) gives the best feasible solution

3

$(x_1, x_2) = (2, 2)$

Let S_1 and S_2 be the unused daily amounts of M1 and M2.

For M1: $S_1 = 24 - (6x_1 + 4x_2) = 4$ tons/day

For M2: $S_2 = 6 - (x_1 + 2x_2)$

$= 6 - (2 + 2 \times 2) = 0$ tons/day

4

Quantity discount results in the following nonlinear objective function:

$$Z = \begin{cases} 5x_1 + 4x_2, & 0 \leq x_1 \leq 2 \\ 4.5x_1 + 4x_2, & x_1 > 2 \end{cases}$$

The situation cannot be treated as a linear program. Nonlinearity can be accounted for in this case using mixed integer programming (chapter 9).