Chapter 1

WALKING

1.1. Ballots and Preference Schedules

1.

Number of voters	**5**	**3**	**5**	**3**	**2**	**3**
1st choice	*A*	*A*	*C*	*D*	*D*	*B*
2nd choice	*B*	*D*	*E*	*C*	*C*	*E*
3rd choice	*C*	*B*	*D*	*B*	*B*	*A*
4th choice	*D*	*C*	*A*	*E*	*A*	*C*
5th choice	*E*	*E*	*B*	*A*	*E*	*D*

This schedule was constructed by noting, for example, that there were five ballots listing candidate *C* as the first preference, candidate *E* as the second preference, candidate *D* as the third preference, candidate *A* as the fourth preference, and candidate *B* as the last preference.

2.

Number of voters	**4**	**5**	**6**	**2**
1st choice	*A*	*B*	*C*	*A*
2nd choice	*D*	*C*	*A*	*C*
3rd choice	*B*	*D*	*D*	*D*
4th choice	*C*	*A*	*B*	*B*

3. **(a)** 5 + 5 + 3 + 3 + 3 + 2 = 21

(b) 11. There are 21 votes all together. A majority is more than half of the votes, or at least 11.

(c) Chavez. Argand has 3 last-place votes, Brandt has 5 last-place votes, Chavez has no last-place votes, Dietz has 3 last-place votes, and Epstein has 5 + 3 + 2 = 10 last-place votes.

4. **(a)** 202 + 160 + 153 + 145 + 125 + 110 + 108 + 102 + 55 = 1160

(b) 581; There are 1160 votes all together. A majority is more than half of the votes, or at least 581.

(c) Alicia. She has no last-place votes. Note that Brandy has 125 + 110 + 55 = 290 last-place votes, Cleo has 202 + 145 + 102 = 449 last-place votes, and Dionne has 160 + 153 + 108 = 421 last-place votes.

5.

Number of voters	**37**	**36**	**24**	**13**	**5**
1st choice	*B*	*A*	*B*	*E*	*C*
2nd choice	*E*	*B*	*A*	*B*	*E*
3rd choice	*A*	*D*	*D*	*C*	*A*
4th choice	*C*	*C*	*E*	*A*	*D*
5th choice	*D*	*E*	*C*	*D*	*B*

Here Brownstein was listed first by 37 voters. Those same 37 voters listed Easton as their second choice, Alvarez as their third choice, Clarkson as their fourth choice, and Dax as their last choice.

6.

Number of voters	**14**	**10**	**8**	**7**	**4**
1st choice	*B*	*B*	*A*	*D*	*E*
2nd choice	*A*	*D*	*B*	*C*	*B*
3rd choice	*E*	*A*	*E*	*B*	*A*
4th choice	*C*	*E*	*D*	*E*	*C*
5th choice	*D*	*C*	*C*	*A*	*D*

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7.

Number of voters	14	10	8	7	4
A	2	3	1	5	3
B	1	1	2	3	2
C	5	5	5	2	4
D	4	2	4	1	5
E	3	4	3	4	1

Here 14 voters had the same preference ballot listing B as their first choice, A as their second choice, E as their third choice, D as their fourth choice, and C as their fifth and last choice.

8.

Number of voters	37	36	24	13	5
A	1	2	5	2	4
B	3	1	2	4	1
C	2	4	3	1	5
D	5	3	1	5	2
E	4	5	4	3	3

9.

Number of voters	255	480	765
1st choice	L	C	M
2nd choice	M	M	L
3rd choice	C	L	C

(0.17)(1500) = 255; (0.32)(500) = 480; The remaining voters (51% of 1500 or 1500-255-480=765) prefer M the most, C the least, so that L is their second choice.

10.

Number of voters	450	900	225	675
1st choice	A	B	C	C
2nd choice	C	C	B	A
3rd choice	B	A	A	B

100% - 20% - 40% = 40% of the voters number 225 + 675 = 900. So, if N represents the total number of voters, then $(0.40)N = 900$. This means there are $N = 2250$ total voters. 20% of 2250 is 450 (these voters have preference ballots A, C, B). 40% of 2250 is 900 (these voters have preference ballots B, C, A).

1.2. Plurality Method

11. **(a)** C. A has 15 first-place votes. B has 11 + 8 + 1 = 20 first-place votes. C has 27 first-place votes. D has 9 first-place votes. C has the most first-place votes with 27 and wins the election.

(b) C, B, A, D. Candidates are ranked according to the number of first-place votes they received (27, 20, 15, and 9 for C, B, A, and D respectively).

12. **(a)** D. A has 21 first-place votes. B has 18 first-place votes. C has 10 + 1 = 11 first-place votes. D has 29 first-place votes. D has the most first-place votes with 29 and wins the election.

(b) D, A, B, C.

13. **(a)** C. A has 5 first-place votes. B has 4 + 2 = 6 first-place votes. C has 6 + 2 + 2 + 2 = 12 first-place votes. D has no first-place votes. C has the most first-place votes with 12 and wins the election.

(b) C, B, A, D. Candidates are ranked according to the number of first-place votes they received (12, 6, 5, and 0 for C, B, A, and D respectively).

14. **(a)** B. A has 6 + 3 = 9 first-place votes. B has 6 + 5 + 3 = 14 first-place votes. C has no first-place votes. D has 4 first-place votes. B has the most first-place votes with 14 and wins the election.

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(b) *B, A, D, C*. Candidates are ranked according to the number of first-place votes they received (14, 9, 4 and 0 for *B, A, D*, and *C* respectively).

15. (a) *D*. *A* has 11% of the first-place votes. *B* has 14% of the first-place votes. *C* has 24% of the first-place votes. *D* has 23% + 19% + 9% = 51% of the first-place votes. *E* has no first-place votes. *D* has the largest percentage of first-place votes with 51% and wins the election.

(b) *D, C, B, A, E*. Candidates are ranked according to the percentage of first-place votes they received (51%, 24%, 14%, 11% and 0% for *D, C, B, A*, and *E* respectively).

16. (a) *C*. *A* has 12% of the first-place votes. *B* has 15% of the first-place votes. *C* has 25% + 10% + 9% + 8% = 52% of the first-place votes. *D* has no first-place votes. *E* has 21% of the first-place votes. *C* has the largest percentage of first-place votes with 52% and wins the election.

(b) *C, E, B, A, D*. Candidates are ranked according to the percentage of first-place votes they received (52%, 21%, 15%, 12%, and 0% for *C, E, B, A* and *D* respectively).

17. (a) *A*. *A* has 5 + 3 = 8 first-place votes. *B* has 3 first-place votes. *C* has 5 first-place votes. *D* has 3 + 2 = 5 first-place votes. *E* has no first-place votes. *A* has the most first-place votes with 8 and wins the election.

(b) *A, C, D, B, E*. Candidates are ranked according to the number of first-place votes they received (8, 5, 5, 3, and 0 for *A, C, D, B*, and *E* respectively). Since both candidates *C* and *D* have 5 first-place votes, the tie in ranking is broken by looking at last-place votes. Since *C* has no last-place votes and *D* has 3 last-place votes, candidate *C* is ranked above candidate *D*.

18. (a) *A*. *A* has 153 + 102 + 55 = 310 first-place votes. *B* has 202 + 108 = 310 first-place votes. *C* has 160 + 110 = 270 first-place votes. *D* has 145 + 125 = 270 first-place votes. Both *A* and *B* have the most first-place votes with 310 so the tie is broken using last-place votes. *A* has no last-place votes. *B* has 125 + 110 + 55 = 290 last-place votes. So *A* wins the election.

(b) *A, B, D, C*. Candidates are ranked according to the number of first-place votes they received (310, 310, 270 and 270 for *A, B, D*, and *C* respectively). In part (a), we saw that the tie between *A* and *B* was broken in favor of *A*. Since both candidates *C* and *D* have 270 first-place votes, the tie in ranking is broken by looking at last-place votes. Since *C* has 202 + 145 + 102 = 449 last-place votes and *D* has 160 + 153 + 108 = 421 last-place votes, candidate *D* is ranked above candidate *C*.

19. (a) *A*. *A* has 5 + 3 = 8 first-place votes. *B* has 3 first-place votes. *C* has 5 first-place votes. *D* has 3 + 2 = 5 first-place votes. *E* has no first-place votes. *A* has the most first-place votes with 8 and wins the election. (Note: This is exactly the same as in Exercise 17(a).)

(b) *A, C, D, B, E*. Candidates are ranked according to the number of first-place votes they received (8, 5, 5, 3, and 0 for *A, C, D, B*, and *E* respectively). Since both candidates *C* and *D* have 5 first-place votes, the tie in ranking is broken by a head-to-head comparison between the two. But candidate *C* is ranked higher than *D* on 5 + 5 + 3 = 13 of the 21 ballots (a majority). Therefore, candidate *C* is ranked above candidate *D*.

20. (a) *B*. *A* has 153 + 102 + 55 = 310 first-place votes. *B* has 202 + 108 = 310 first-place votes. *C* has 160 + 110 = 270 first-place votes. *D* has 145 + 125 = 270 first-place votes. Both *A* and *B* have the most first-place votes with 310 so the tie is broken by head-to-head comparison. But candidate *A* is ranked higher than *B* on 153 + 125 + 110 + 102 + 55 = 545 of the 1160 ballots (less than a majority). So *B* wins the tiebreaker and the election.

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(b) *B, A, D, C.* Candidates are ranked according to the number of first-place votes they received (310, 310, 270 and 270 for *B*, *A*, *D*, and *C* respectively). In part (a), we saw that the tie between *A* and *B* was broken in favor of *B*. Since both candidates *C* and *D* have 270 first-place votes, the tie in ranking is broken by head-to-head comparison. Candidate *C* is ranked higher than *D* on 160 + 153 + 110 + 108 = 531 of the 1160 ballots (less than a majority). So in the final ranking, candidate *D* is ranked above candidate *C*.

1.3. Borda Count

21. **(a)** *A* has $4\times 15+3\times(9+8+1)+2\times 11+1\times 27=163$ points.
B has $4\times(11+8+1)+3\times 15+2\times(27+9)+1\times 0=197$ points.
C has $4\times 27+3\times 0+2\times 8+1\times(15+11+9+1)=160$ points.
D has $4\times 9+3\times(27+11)+2\times(15+1)+1\times 8=190$ points.
The winner is *B*.

(b) *B*, *D, A*, *C.* Candidates are ranked according to the number of Borda points they received.

22. **(a)** *A* has $4\times 21+3\times 18+2\times(29+10)+1\times 1=217$ points.
B has $4\times 18+3\times(10+1)+2\times 21+1\times 29=176$ points.
C has $4\times(10+1)+3\times(29+21)+2\times 18+1\times 0=230$ points.
D has $4\times 29+3\times 0+2\times 1+1\times(21+18+10)=167$ points.
The winner is *C*.

(b) *C, A, B, D.* Candidates are ranked according to the number of Borda points they received.

23. **(a)** *A* has $4\times 5+3\times 2+2\times(6+2)+1\times(4+2+2)=50$ points.
B has $4\times(4+2)+3\times(2+2)+2\times 2+1\times(6+5)=51$ points.
C has $4\times(6+2+2+2)+3\times 0+2\times(5+4+2)+1\times 0=70$ points.
D has $4\times 0+3\times(6+5+4+2)+2\times 2+1\times(2+2)=59$ points.
The winner is *C*.

(b) *C, D, B, A.* Candidates are ranked according to the number of Borda points they received.

24. **(a)** *A* has $4\times(6+3)+3\times(4+3)+2\times 6+1\times 5=74$ points.
B has $4\times(6+5+3)+3\times 3+2\times 0+1\times(6+4)=75$ points.
C has $4\times 0+3\times(6+6+5)+2\times(4+3+3)+1\times 0=71$ points.
D has $4\times 4+3\times 0+2\times(6+5)+1\times(6+3+3)=50$ points.
The winner is B.

(b) *B, A, C, D.* Candidates are ranked according to the number of Borda points they received.

25. Here we can use a total of 100 voters for simplicity.
A has $5\times 11+4\times(24+23+19)+3\times(14+9)+2\times 0+1\times 0=388$ points.
B has $5\times 14+4\times 0+3\times(24+11)+2\times 23+1\times(19+9)=249$ points.
C has $5\times 24+4\times(14+11+9)+3\times 23+2\times 19+1\times 0=363$ points.
D has $5\times(23+19+9)+4\times 0+3\times 0+2\times 14+1\times(24+11)=318$ points.
E has $5\times 0+4\times 0+3\times 19+2\times(24+11+9)+1\times(23+14)=182$ points.
The ranking (according to Borda points) is *A, C, D, B, E*.

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