CHAPTER 1

1.1. (a) Total distance = $1 + \frac{1}{2} + \frac{1}{4} + \cdots = \frac{1}{1-\frac{1}{2}} = 2$ m

(b) Distance north = $1 - \frac{1}{4} + \frac{1}{16} - \cdots = \frac{1}{1+\frac{1}{4}} = 0.8$ m

Distance east = $\frac{1}{2} - \frac{1}{8} + \frac{1}{32} - \cdots = \frac{1}{2}\left(1 - \frac{1}{4} + \frac{1}{16} - \cdots\right) = 0.4$ m

$\therefore$ Final position is (0.8, 0.4)

(c) Straight line distance = $\sqrt{(0.8)^2 + (0.4)^2} = 0.8944$ m

1.2. $\mathbf{A} + \mathbf{B} + \mathbf{C} = 2\mathbf{a}_1 + 3\mathbf{a}_2 + 2\mathbf{a}_3$ — (1)

$2\mathbf{A} + \mathbf{B} - \mathbf{C} = \mathbf{a}_1 + 3\mathbf{a}_2$ — (2)

$\mathbf{A} - 2\mathbf{B} + 3\mathbf{C} = 4\mathbf{a}_1 + 5\mathbf{a}_2 + \mathbf{a}_3$ — (3)

(1) + (2) $\rightarrow 3\mathbf{A} + 2\mathbf{B} = 3\mathbf{a}_1 + 16\mathbf{a}_2 + 2\mathbf{a}_3$ — (4)

(2) $\times$ 3 + (3) $\rightarrow 7\mathbf{A} + \mathbf{B} = 7\mathbf{a}_1 + 14\mathbf{a}_2 + \mathbf{a}_3$ — (5)

[(5) $\times$ 2 – (4)] $\div$ 11 $\rightarrow \mathbf{A} = \mathbf{a}_1 + 2\mathbf{a}_2$ — (6)

(5) – (6) $\times$ 7 $\rightarrow$ $\mathbf{B} = \mathbf{a}_3$ — (7)

(1) – (6) – (7) $\rightarrow$ $\mathbf{C} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$ — (8)

1.3. $(\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B}) = \mathbf{A} \cdot \mathbf{A} - \mathbf{A} \cdot \mathbf{B} + \mathbf{B} \cdot \mathbf{A} - \mathbf{B} \cdot \mathbf{B} = A^2 - B^2$

$(\mathbf{A} + \mathbf{B}) \times (\mathbf{A} - \mathbf{B}) = \mathbf{A} \times \mathbf{A} - \mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A} - \mathbf{B} \times \mathbf{B} = 2\mathbf{B} \times \mathbf{A}$

For $\mathbf{A} = 3\mathbf{a}_1 - 5\mathbf{a}_2 + 4\mathbf{a}_3$ and $\mathbf{B} = \mathbf{a}_1 + \mathbf{a}_2 - 2\mathbf{a}_3$,

$\mathbf{A} + \mathbf{B} = 4\mathbf{a}_1 - 4\mathbf{a}_2 + 2\mathbf{a}_3$, $\mathbf{A} - \mathbf{B} = 2\mathbf{a}_1 - 6\mathbf{a}_2 + 6\mathbf{a}_3$,

$A^2 = 9 + 25 + 16 = 50$, and $B^2 = 1 + 1 + 4 = 6$

$(\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B}) = 8 + 24 + 12 = 44 = A^2 - B^2$

$$(\mathbf{A}+\mathbf{B})\times(\mathbf{A}-\mathbf{B}) = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 4 & -4 & 2 \\ 2 & -6 & 6 \end{vmatrix} = -12\mathbf{a}_x - 20\mathbf{a}_y - 16\mathbf{a}_z$$

$$= 2\begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 1 & 1 & -2 \\ 3 & -5 & 4 \end{vmatrix} = 2\mathbf{B}\times\mathbf{A}$$

1.4. $\mathbf{B}\times\mathbf{C} = -4\mathbf{a}_x + 2\mathbf{a}_y + 8\mathbf{a}_z$, $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = 8\mathbf{a}_x + 16\mathbf{a}_y$

$\mathbf{C}\times\mathbf{A} = -\mathbf{a}_x - 2\mathbf{a}_y + 7\mathbf{a}_z$, $\mathbf{B}\times(\mathbf{C}\times\mathbf{A}) = -12\mathbf{a}_x - 8\mathbf{a}_y - 4\mathbf{a}_z$

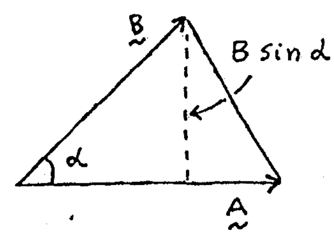
$\mathbf{A}\times\mathbf{B} = \mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$, $\mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = 4\mathbf{a}_x - 8\mathbf{a}_y + 4\mathbf{a}_z$

$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = 0$

In fact, this quantity is zero for any **A**, **B**, and **C**.

1.5. Area $= \frac{1}{2}AB\sin\alpha = \frac{1}{2}|\mathbf{A}\times\mathbf{B}|$

For the points (1, 2, 1), (–3, –4, 5), and (2, –1, –3),



$\mathbf{A} = 4\mathbf{a}_x + 6\mathbf{a}_y - 4\mathbf{a}_z$

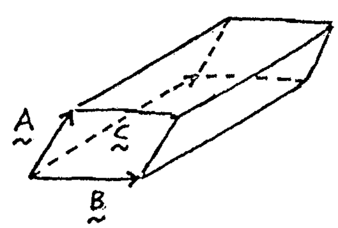
$\mathbf{B} = 5\mathbf{a}_x + 3\mathbf{a}_y - 8\mathbf{a}_z$

$\mathbf{A}\times\mathbf{B} = -36\mathbf{a}_x + 12\mathbf{a}_y - 18\mathbf{a}_z$

$\therefore$ Area $= \frac{1}{2}\sqrt{(-36)^2 + (12)^2 + (-18)^2} = 21$ units.

1.6. Area of the base $= |\mathbf{B}\times\mathbf{C}|$

Height of parallelepiped = Projection of **A** onto the normal to the base



$= \mathbf{A}\cdot\dfrac{\mathbf{B}\times\mathbf{C}}{|\mathbf{B}\times\mathbf{C}|}$

$\therefore$ Volume of parallelepiped = Area of base × height = $\mathbf{A}\cdot\mathbf{B}\times\mathbf{C}$

For $\mathbf{A} = 4\mathbf{a}_x$, $\mathbf{B} = 2\mathbf{a}_x + \mathbf{a}_y + 3\mathbf{a}_z$, and $\mathbf{C} = 2\mathbf{a}_y + 6\mathbf{a}_z$, $\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = 0$.

Hence, volume of the parallelepiped is zero. The three vectors lie in a plane.

1.7. The vector **A** must be perpendicular to both $(-\mathbf{a}_y + 2\mathbf{a}_z)$ and $(\mathbf{a}_x - 2\mathbf{a}_z)$.

Hence $\mathbf{A} = C(-\mathbf{a}_y + 2\mathbf{a}_z) \times (\mathbf{a}_x - 2\mathbf{a}_z) = C(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)$ where C is a constant. To find C,

we note that $\mathbf{a}_x \times \mathbf{A} = \mathbf{a}_x \times C(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z) = 2\mathbf{a}_z - \mathbf{a}_y$

$\therefore\ C = 1$ and $\mathbf{A} = 2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$.

Verification: $\mathbf{a}_y \times \mathbf{A} = \mathbf{a}_y \times (2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z) = \mathbf{a}_x - 2\mathbf{a}_z$.

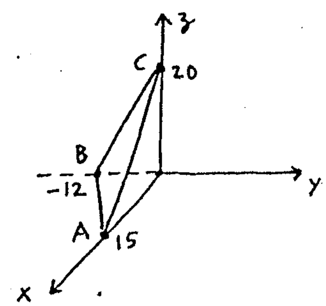
1.8. Vector from $A(5, 0, 3)$ to $B(3, 3, 2) = -2\mathbf{a}_x + 3\mathbf{a}_y - \mathbf{a}_z$

Vector from $C(6, 2, 4)$ to $D(3, 3, 6) = -3\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z$

Component of **AB** along **CD** = $\mathbf{AB} \cdot \dfrac{\mathbf{CD}}{CD} = \dfrac{6+3-2}{\sqrt{9+1+4}} = 1.8708$

1.9. Writing the equation for the plane as $\dfrac{x}{15} - \dfrac{y}{12} + \dfrac{z}{20} = 1$, we find the intercepts on the x, y,

and z-axes to be at 15, –12, and 20, respectively. Thus



$\mathbf{R}_{AB} = -15\mathbf{a}_x - 12\mathbf{a}_y$

$\mathbf{R}_{AC} = -15\mathbf{a}_x + 20\mathbf{a}_z$

$\mathbf{R}_{AC} \times \mathbf{R}_{AB} = 240\mathbf{a}_x - 300\mathbf{a}_y + 180\mathbf{a}_z$

$$\mathbf{a}_n = \frac{\mathbf{R}_{AC} \times \mathbf{R}_{AB}}{|\mathbf{R}_{AC} \times \mathbf{R}_{AB}|} = \frac{4\mathbf{a}_x - 5\mathbf{a}_y + 3\mathbf{a}_z}{5\sqrt{2}}$$

Distance from origin to the plane = $15\mathbf{a}_x \cdot \mathbf{a}_n = 6\sqrt{2}$.

1.10. For $y = 2x$, $z = 4y$, we have $dy = 2\,dx$, $dz = 4\,dy = 8\,dx$.

$\therefore\ d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z = dx\,\mathbf{a}_x + 2\,dx\,\mathbf{a}_y + 8\,dx\,\mathbf{a}_z$

$= (\mathbf{a}_x + 2\mathbf{a}_y + 8\mathbf{a}_z)\,dx$, independent of the point.

1.11. For $x = y = z^2$, we have $dx = dy = 2z\,dz$.

At the point (4, 4, 2), $dx = dy = 4\,dz$

$\therefore\ d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z = 4\,dz\,\mathbf{a}_x + 4\,dz\,\mathbf{a}_y + dz\,\mathbf{a}_z$

$= (4\mathbf{a}_x + 4\mathbf{a}_y + \mathbf{a}_z)\,dz$