Section 2.2

In this edition we present the traditional Gauss-Jordan method as Method 1 in Example 1, and the improved version as Method 2. Our experience is that students are much more successful in carrying out the improved version, so we present it as the first method in Example 2 and the only method in later examples. As we suggest in Example 2, the traditional method is appropriate when technology is available.

Shown below is a comparison between the improved version and the traditional Gauss-Jordan method. Note the absence of tedious fractions in the improved version.

Solve:

$$\begin{aligned} 4x - 2y - 3z &= -23 \\ -4x + 3y + z &= 11 \\ 8x - 5y + 4z &= 6 \end{aligned}$$

New Method

$$\left[\begin{array}{rrr|r} 4 & -2 & -3 & -23 \\ -4 & 3 & 1 & 11 \\ 8 & -5 & 4 & 6 \end{array}\right]$$

$$\begin{array}{r} \\ R_1 + R_2 \to R_2 \\ -2R_1 + R_3 \to R_3 \end{array} \left[\begin{array}{rrr|r} 4 & -2 & -3 & -23 \\ 0 & 1 & -2 & -12 \\ 0 & -1 & 10 & 52 \end{array}\right]$$

$$\begin{array}{r} R_1 + 2R_2 \to R_1 \\ \\ R_2 + R_3 \to R_3 \end{array} \left[\begin{array}{rrr|r} 4 & 0 & -7 & -47 \\ 0 & 1 & -2 & -12 \\ 0 & 0 & 8 & 40 \end{array}\right]$$

$$\begin{array}{r} 7R_3 + 8R_1 \to R_1 \\ R_3 + 4R_2 \to R_2 \\ \\ \end{array} \left[\begin{array}{rrr|r} 32 & 0 & 0 & -96 \\ 0 & 4 & 0 & -8 \\ 0 & 0 & 8 & 40 \end{array}\right]$$

$$\begin{array}{r} \frac{1}{32}R_1 \to R_1 \\ \frac{1}{4}R_2 \to R_2 \\ \frac{1}{8}R_3 \to R_3 \end{array} \left[\begin{array}{rrr|r} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$(-3, -2, 5)$ is the solution.

Traditional Method

$$\left[\begin{array}{rrr|r} 4 & -2 & -3 & -23 \\ -4 & 3 & 1 & 11 \\ 8 & -5 & 4 & 6 \end{array}\right]$$

$$\begin{array}{r} \frac{1}{4}R_1 \to R_1 \\ \\ \\ \end{array} \left[\begin{array}{rrr|r} 1 & -\frac{1}{2} & -\frac{3}{4} & -\frac{23}{4} \\ -4 & 3 & 1 & 11 \\ 8 & -5 & 4 & 6 \end{array}\right]$$

$$\begin{array}{r} \\ 4R_1 + R_2 \to R_2 \\ -8R_1 + R_3 \to R_3 \end{array} \left[\begin{array}{rrr|r} 1 & -\frac{1}{2} & -\frac{3}{4} & -\frac{23}{4} \\ 0 & 1 & -2 & -12 \\ 0 & -1 & 10 & 52 \end{array}\right]$$

$$\begin{array}{r} \frac{1}{2}R_2 + R_1 \to R_1 \\ \\ R_2 + R_3 \to R_3 \end{array} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{7}{4} & -\frac{47}{4} \\ 0 & 1 & -2 & -12 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \frac{1}{8}R_3 \to R_3 \end{array} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{7}{4} & -\frac{47}{7} \\ 0 & 1 & -2 & -12 \\ 0 & 0 & 8 & 40 \end{array}\right]$$

$$\begin{array}{r} \\ \frac{7}{4}R_3 + R_1 \to R_1 \\ 2R_3 + R_2 \to R_2 \end{array} \left[\begin{array}{rrr|r} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$(-3, -2, 5)$ is the solution.

By reworking a problem from Section 2.1 using the Gauss-Jordan method, students will see how closely this method parallels the echelon method given there.

Remind students to operate on the entire row. A common error is to forget the entry to the right of the vertical bar.

Section 2.3

Mention that, as in algebra, only like things can be added or subtracted. In this case, the like things are matrices having the same dimensions.

Section 2.4

Using the visual approach to matrix multiplication given in Examples 2 and 3, students will have no trouble multiplying matrices. Most will eventually no longer need this tool.

Section 2.5

Explain that the technique used in finding the multiplicative inverse of a matrix is still the Gauss-Jordan method, now with more than one entry per row to the right of the vertical bar.

Students may be resistant to learning another method for solving a system of equations. Stress the advantage of using the inverse method to solve systems having the same matrix of coefficients. See Example 5. Point out that these systems can be found in many different fields of application. See Exercise 60.

Section 2.6

Discuss how the entries of A, the input-output matrix, could be determined. Stress the economic significance of the matrices A, D, X, and AX.

Section 3.1

Emphasize that the test point can be *any* point *not* on the boundary. Choose several points on either side of the boundary and on the boundary itself to illustrate this concept.

Students may fall into the habit of always choosing (0, 0) as the test point. Do a couple of problems where (0, 0) is not available for use as a test point.

Using a different color to shade each half plane for a system of inequalities will make their overlap easier to recognize.

Section 3.2

Use diagrams like Figures 12 and 13 to convince students of the believability of the corner point theorem. Emphasize that a corner point must be a point in the feasible region. Also, stress that not all corner points can be found by inspection. Some require solving a system of two linear equations. Have students note the equation of the boundary line next to its graph, so they will know which equations to solve as a system.

Section 3.3

Review the guidelines for setting up an applied problem (Section 2.1) to determine the objective function and all necessary constraints.

Students find those constraints comparing two unknown quantities the most difficult. See Exercise 12 for an example of this type of constraint.

Section 4.1

The simplex method in this chapter is modified from the traditional method along the lines of the Gauss-Jordan method in Chapter 2, eliminating tedious fractions until the last step. The notation of s instead of x for slack and surplus variables will help students remember which variables are the originals and which are slack or surplus variables.

Note the horizontal line in the simplex tableau to separate the constraints from the objective function.

Students may need several examples to be able to pick out the basic variables and to find the basic feasible solution from a matrix.